meso-SAS, a modification of the SAS for meso-scale models

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Problem with the conventional mass flux schemes

- Most of the mass-flux schemes are based on the original Arakawa-Schubert (1974) assumption that the updraft area is much smaller than the model grid size. This assumption begins to break down when the grid sizes become smaller than 10 km. Since the assumption is fundamental to the parameterization scheme, we are not justified to continue to use such schemes.

Should we go directly to the explicit schemes?

- While the commonly used mass flux schemes should be avoided when σ (the ratio of the updraft area to the grid area) is no longer small, the use of the explicit microphysics scheme is still problematic since the vertical motion in models of grid sizes from 500m to 10 km may not be large enough to smoothly create moist adiabat for the entire grid point. This can and do leads to the so-called grid-point storm when computational instability can lead to excess rainfall and much lower surface pressure for hurricanes.

So when can we stop parameterizing moist convection?

- When $\sigma < .1$, we can safely use the conventional mass flux schemes.
- When $\sigma > .9$, we can most likely use explicit microphysics directly and skip the parameterization.
- When $.1 < \sigma < .9$, we are in no-man's land. We need to parameterize the convection but we can not use the conventional scheme.

Proposing a modification of the A-S scheme

- We have re-derived the A-S scheme removing the assumption that the updraft area be small.
- In doing so, we have arrived at a scheme that can be easily implemented.
- It is similar to the conventional scheme when the updraft area is small.
- Its effect diminishes when the updraft area approaches the grid area (convergence issue).
- It explicitly takes the updraft area into consideration.

Assuming that the properties of the atmosphere inside the updraft area (as well as outside, though at different values) are uniform, we can define the grid-mean properties as a linear combination of the properties in and out of the updraft area σ :

$$\overline{s} = \sigma s_c + (1 - \sigma)\widetilde{s}$$

$$\overline{w} = \sigma w_c + (1 - \sigma)\widetilde{w}$$

Where s is the static energy and w the vertical velocity. The subscript c denotes the updraft fraction and the tilde denotes the 'environment'.

The sub-grid vertical transport of the dry static energy can be expressed as:

$$\overline{w's'} = \sigma_c(w_c - \overline{w})(s_c - \overline{s}) + (1 - \sigma_c)(\widetilde{w} - \overline{w})(\widetilde{s} - \overline{s})$$

$$= \sigma_c w_c s_c + \sigma_c \overline{w}\,\overline{s} - \sigma_c w_c \overline{s} - \sigma_c \overline{w} s_c$$

$$+ (1 - \sigma_c)[\widetilde{w}\,\widetilde{s} + \overline{w}\,\overline{s} - \widetilde{w}\,\overline{s} - \overline{w}\,\widetilde{s}]$$

$$= \overline{w}\,\overline{s} + \sigma_c w_c(s_c - \overline{s}) - \sigma_c \overline{w}\,s_c$$

$$+ (1 - \sigma_c)\widetilde{w}(\widetilde{s} - \overline{s}) - (1 - \sigma_c)\overline{w}\,\widetilde{s}$$

$$= \sigma_c w_c(s_c - \overline{s}) + (1 - \sigma_c)\widetilde{w}(\widetilde{s} - \overline{s})$$

It can be seen that when σ_c approaches 1, the transport would go to zero.

When we define the mass flux in the following way:

$$M_c = \rho \sigma_c w_c, \ \widetilde{M} = \rho(1 - \sigma_c)\tilde{w}, \ \text{and} \ \rho\overline{w} = M_c + \widetilde{M}$$

The sub-grid scale transport can be expressed in the form of the mass flux:

$$\overline{\rho w's'} = M_c(s_c - \overline{s}) + \widetilde{M}(\tilde{s} - \overline{s})$$

The sub-grid scale heating and drying effect can be expressed as:

$$\frac{\partial s}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(s_c - \overline{s}) + (\rho\overline{w} - M_c)(\tilde{s} - \overline{s})] + [Lc]_{conv}$$

$$\frac{\partial q}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_c - \overline{q}) + (\rho\overline{w} - M_c)(\tilde{q} - \overline{q})] - [c]_{conv}$$

The moist static energy equation can be derived by combining the above two equations as:

$$\frac{\partial h}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(h_c - \overline{h}) + (\rho\overline{w} - M_c)(\tilde{h} - \overline{h})]$$

We will re-arrange the terms:

$$\frac{\partial h}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(h_c - \tilde{h}) + \rho\overline{w}(\tilde{h} - \overline{h})]$$

$$\frac{\partial q}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_c - \tilde{q}) + \rho\overline{w}(\tilde{q} - \overline{q})] - [c]_{conv}$$

$$\frac{\partial q_l}{\partial t}\bigg|_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_l^c - \tilde{q_l}) + \rho\overline{w}(\tilde{q_l} - \overline{q_l})] + [c - rain]_{conv}$$

Following A-S, we introduce a static cloud model as:

$$-\frac{\partial M_c}{\partial z} + E - D = 0$$

$$-\frac{\partial}{\partial z}[M_c s_c] + E\tilde{s} - Ds_c + Lc = 0$$

$$-\frac{\partial}{\partial z}[M_c q_c] + E\tilde{q} - Dq_c - c = 0$$

$$-\frac{\partial}{\partial z}[M_c q_l^c] + E\tilde{q_l} - Dq_l^c + c - rain = 0$$

$$-\frac{\partial}{\partial z}[M_c h_c] + E\tilde{h} - Dh_c = 0$$

Note the use of the environmental properties in the entrainment terms. This portion of the derivation may give us trouble when the updraft area approaches one. We will work on improving it at a later time.

The actual equations used in the meso-SAS is as follows:

$$\left.\frac{\partial \overline{h}}{\partial t}\right|_{conv} = -\frac{1}{\rho}(E\widetilde{h} - Dh_c) + \frac{1}{\rho}\frac{\partial}{\partial z}[M_c\widetilde{h} + \rho\overline{w}(\overline{h} - \widetilde{h})]$$

$$\left.\frac{\partial \overline{q}}{\partial t}\right|_{conv} = -\frac{1}{\rho}(E\widetilde{q} - Dq_c) + \frac{1}{\rho}\frac{\partial}{\partial z}(M_c\widetilde{q} + \rho\overline{w}(\overline{q} - \widetilde{q})]$$

$$\left.\frac{\partial \overline{q_l}}{\partial t}\right|_{conv} = -\frac{1}{\rho}(E\widetilde{q_l} - Dq_l^c) + \frac{1}{\rho}\frac{\partial}{\partial z}(M_c\widetilde{q_l} + \rho\overline{w}(\overline{q_l} - \widetilde{q_l})]$$

The major differences with the original SAS scheme are the inclusion of the environmental effects and the additional terms involving the grid-mean vertical velocity. When the updraft area is small compared to the grid size, this equation approaches the original SAS equations.

Determination of σ

- The meso-SAS scheme requires the specification of a new variable to close the scheme : the fraction of the updraft area in the grid (σ).

- For the first implementation, we have decided to derive σ based on the ratio of the grid mean vertical velocity and the scaled updraft speed (as a function of cloud work function and the cloud base grid-mean velocity) :

$$\sigma = .91 \overline{w} / w_c + .09$$

Discussions on sigma

- The actual compensating subsidence occurs over a large area, so the environmental vertical velocity should be much smaller than the updraft. We have formulated the fraction σ by assuming that the magnitude of the subsidence is 10% of the magnitude of the updraft. The determination of σ is a weak area of the scheme and will be studied further.

Closure

- For closure, we continue to use the modified quasi-equilibrium assumption to calculate the changes in mass flux due to the buoyancy effect.

- This means the exclusion of the terms involving the grid-mean vertical velocity for the closure calculation.

Differences from the SAS

- We need to obtain the fractional area σ. This is done after we calculated the cloud work function.
- We then calculate the environmental properties.
- We re-calculate the static cloud properties.
- In the small time step integration, we only include the buoyancy effects to calculate the warming and drying.
- The change in cloud work function is used to calculate the mass flux (closure).
- Final heating and drying are calculated using the mass flux (using the equations in slide 12).

Convergence

- While the scheme formally converges when the updraft takes up the whole grid, the actual computation can be problematic as the environmental properties may not be defined. This was pointed out clearly by Arakawa in 2011. We currently turn the scheme off when σ exceeds .9. For the 3km HWRF, this is not a problem as most of the times we find σ to be less than .5.

Discussions

- Other than the assumptions of quasi-equilibrium and the static cloud model (same as SAS), the key addition is the estimation of the fraction area σ. A 'characteristic' cloud updraft speed is used to do this. We do not, otherwise, use the cloud updraft speed explicitly.