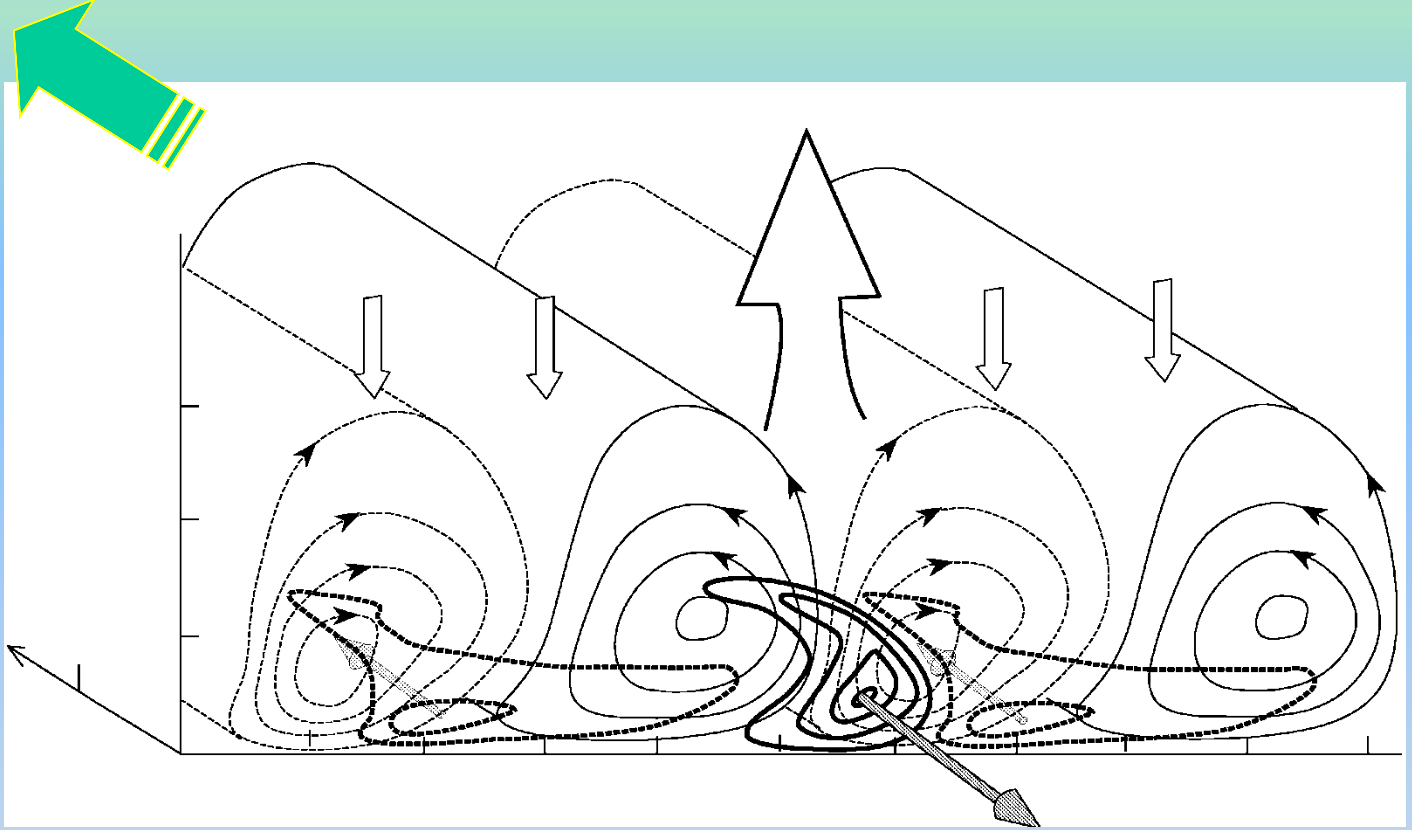
Developing a Parameterization of TCBL roll Vortices

Ralph Foster

APL, University of Washington



Wavelength: Larger-scale structures ~ 700 to 5000 m
Smaller-scale structures ~ 300 to 700 m

Velocity Perturbations: +/- 7 m s^{-1} typical
DOW +/- “10s of” m s^{-1}

Orientation: Typically along-mean TCBL wind, wide variability

Prevalence: Roll-scale structures ~ unknown, (35% to 70%)
Streak-scale structures: ***Most likely usually present***

Roll Effects

- TCBL connects surface fluxes with storm interior
- Largest component is along-roll (roughly along-wind) near-surface wind modulation
- Rolls induce non-local & non-gradient transport of momentum and heat across TCBL

Hypothesis: Roll fluxes are a Significant Unparameterized Feature of TCBLs

- TC Intensity is, in part, related to net compensation between the downward sink of TC momentum into the ocean and the upward flux of enthalpy from the ocean into the TC interior.
- Current emphasis is on ratio of bulk flux coefficients
  - C_D/C_k tends to decrease in high winds, what compensates?
- *Rolls induce inherently non-gradient* (i.e. non-local) downward transport of momentum across the depth of the TCBL
  - Models only parameterize local, down-gradient momentum flux
  - **<u>Is non-gradient flux important to numerical models?</u>**

dicular to the rolls.

Radar volumes were updated every 300 s; these intervals were too long to permit esti-

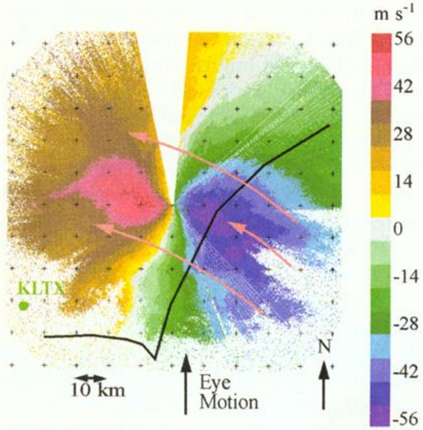


Fig. 3. Large-scale Doppler velocity structure at 23:30:19 UTC, as measured by the DOW radar. Strong easterly flow peaking at ~60 m s^{-1} is evident both off- and onshore. The eye of the hurricane is at the edge of radar visibility to the south. Visibility was severely limited by attenuation. Pink curved arrows illustrate average wind flow. Scan is at 5° elevation.

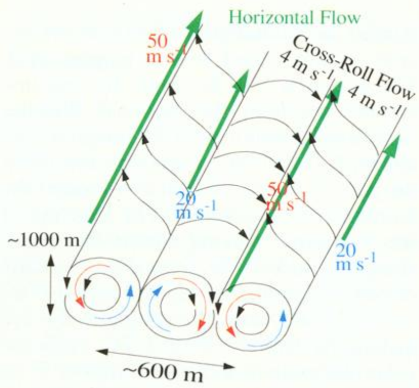


Fig. 5. Schematic representation of observed shear- and wind-parallel boundary layer rolls. High-momentum air (red) is brought to the surface in the downward legs of the rolls, while air slowed near the surface is brought aloft in the upward legs.

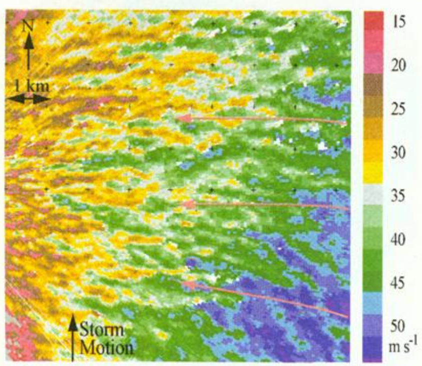


Fig. 4. High-resolution image of Doppler velocity field to the east of Wilmington at 23:58:17 UTC. Sub-kilometer-scale streaks caused by boundary layer rolls modulate the mean easterly flow. Near the radar (left) at altitudes of ~100 m agl, peak and trough wind speed values are ~40 m s^{-1} and ~10 m s^{-1}, respectively. Further from the radar (right), peak and trough wind speed values alternate from ~25 to ~55 m s^{-1}. Azimuthal shear values are (~30 m s^{-1}/~300 m) ≈ 0.1 s^{-1} across many of the rolls. Scan is at 2° elevation.

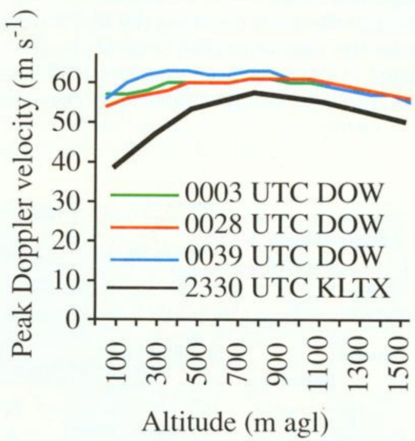


Fig. 6. Altitude dependence of peak wind speeds as observed by DOW and National Weather Service KLTX radars. DOW-measured peak speeds at 100 m agl are nearly as high as those at 1000 m agl as a result of momentum transport in the rolls and agree closely with surface peak wind observations. KLTX-measured peak speeds are smaller at low altitude because of poorer resolution and possibly because of longer overland trajectories.

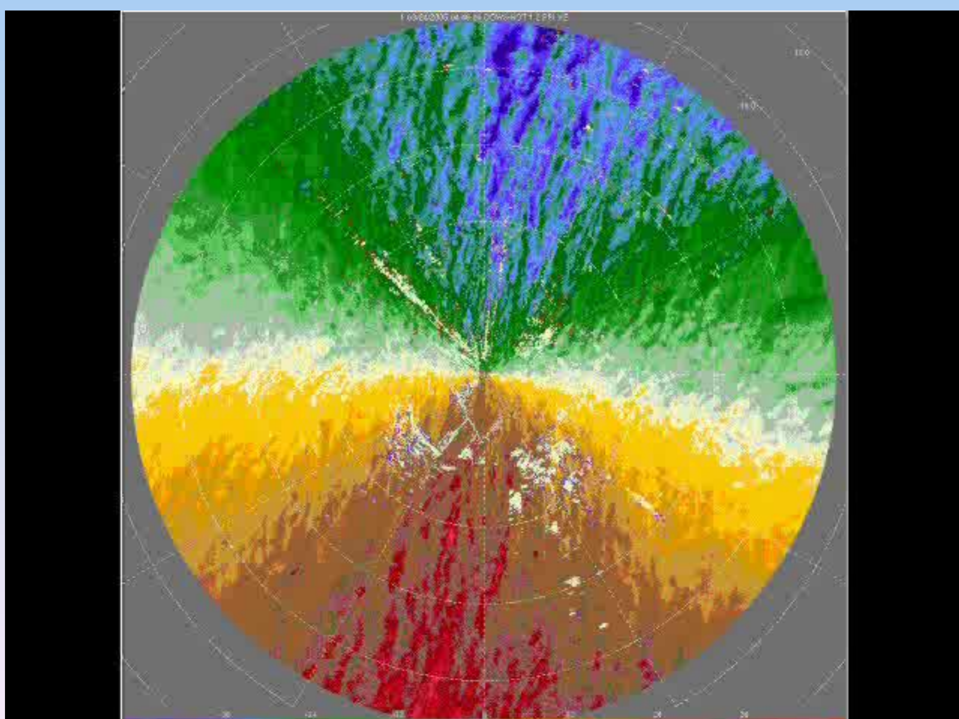
~30 m/s mean +/- 15 m/s across-roll variation in low-level wind

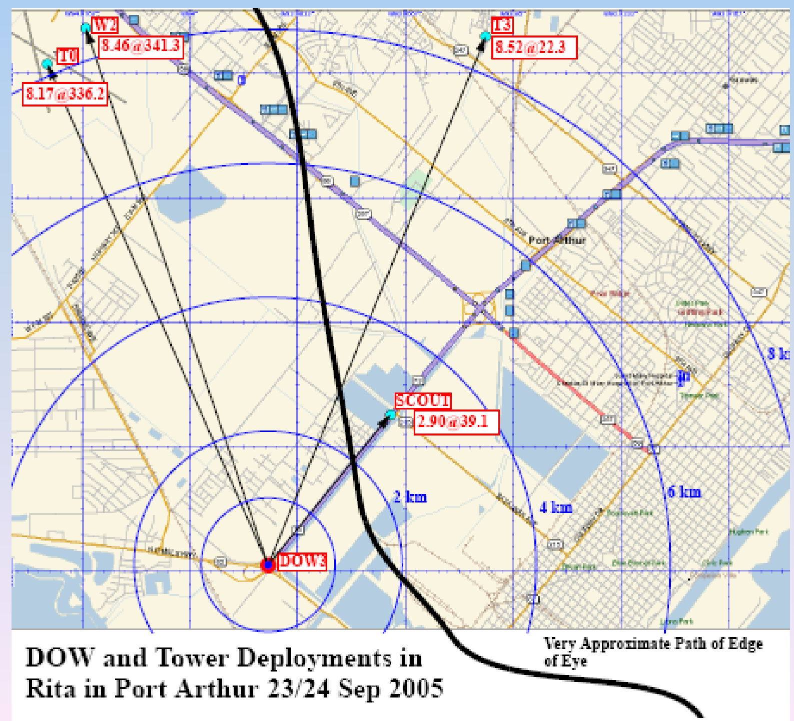
Wurman and Winslow (1998)
Science, **280,** 555-557

Example:

J. Wurman, Doppler on Wheels, Hurricane Rita, 2005

- 1.2° slices every 12 seconds
- Radial Velocity
- Gate Spacing 25 m
- Azimuthal Resolution 0.25°
- 2 km Range Rings (8 km shown)





Parameterization Strategy

- Numerical models use a range of local closures
- Develop roll flux model consistent with existing closures
  - No change to existing parameterizations
  - Added non-gradient flux contribution only if mean flow conditions are consistent with roll formation
- *Use simple theoretical models …*
  - Nonlinear similarity mean TCBL model (local, gradient fluxes)
  - Nonlinear roll stability model
- *… in conjunction with observational data*
  - SAR
  - Doppler Wind Lidar (on NOAA P-3)
  - Radar
  - ??

Scales

Velocity: V_g

Length: $\delta = \sqrt{\dfrac{2K}{I}}$

Temperature: $\Delta T = Tair - Tsfc$

$$I^2 = \left(f + \frac{2V_g}{r}\right)\left(f + \frac{V_g}{r} + \frac{\partial V_g}{\partial r}\right) \quad \text{(inertial stability)}$$

Parameters

$$R_e = \frac{V_g \ldots}{K \ldots}$$

$$Ri = \frac{g}{T_0}\frac{\Delta T \ldots}{V_g^{2} \ldots}$$

$$r_e = \frac{r}{\delta}$$

$$R_o = \frac{V_g}{f\,\delta \ldots}$$

Similarity Assumption

$$U = V_g \; r \; y_1\left(\frac{z}{\delta}\right)$$

$$V = V_g \; r \; y_3\left(\frac{z}{\delta}\right)$$

$$W = \frac{V_g \; r}{r_e \; r} \; y_5\left(\frac{z}{\delta}\right)$$

Nonlinear Mean TCBL Similarity Model

Nonlinear Similarity Equations (ODEs)

$$y_1' = y_2$$

(residual) (residual)

$$y_2' = \frac{R_e}{r_e}\left[\frac{V_p y_1^2 + A\zeta y_1 y_2}{K}\right] - \frac{R_e}{r_e}\frac{y_3^2 - 1}{K} - \frac{R_e}{R_o}\frac{y_3 - 1}{K} + \frac{R_e}{r_e}\frac{y_5 y_2}{K} - \frac{K'}{K} y_2 \, .$$

Radial Advection

Curvature

Coriolis

Vertical Advection

$$y_3' = y_4$$

$$y_4' = \frac{R_e}{r_e}\left[\frac{V_p y_1 y_3 + A\zeta y_1 y_4}{K}\right] + \frac{R_e}{r_e}\frac{y_1 y_3}{K} + \frac{R_e}{R_o}\frac{y_1}{K} + \frac{R_e}{r_e}\frac{y_5 y_4}{K} - \frac{K'}{K} y_4$$

$$y_5' = -y_1 \; V_p + 1 \; - A\zeta y_2 \, .$$

(Plus Temperature equation)

Note: Parameters <u>only</u> appear as ratios (e.g. Re/r_e).
Easy system to solve numerically

Boundary Conditions

$$y_2 \quad 0 \ = \frac{C_D R_e U_0 y_1 \quad 0}{K_0}$$

C_D is Large & Pond, max = 0.0025
(Can use any parameterization)

$$y_4 \quad 0 \ = \frac{C_D R_e U_0 y_3 \quad 0}{K_0} \qquad (constant \ \ flux)$$

Reynolds Number is Key Parameter

$$\lim_{z\to\infty}\begin{cases} y_1 = 0 \\ y_3 = 1. \end{cases}$$

Entrainment Flux at PBL top is easy to implement:
small effect on what follows

"Cross-Flow Instability"

Triggered by instability in cross-stream component of 3D profile

$$i\alpha u + Dw = \frac{1}{r_e}\left(u\sin(\varepsilon) + v\cos(\varepsilon)\right)$$

$$\left[\frac{\partial}{\partial t} + i\alpha U - \frac{1}{R_e}(D^2 - \alpha^2)\right]u + wU' - \frac{1}{R_o}v + i\alpha p + \frac{1}{r_e}\left[-V\cos(\varepsilon)u + (2V\sin(\varepsilon) - U\cos(\varepsilon))v\right] =$$
$$-\left(uD_x + wD\right)u - \frac{1}{r_e}\left(\cos(\varepsilon)u + \sin(\varepsilon)v\right)v$$

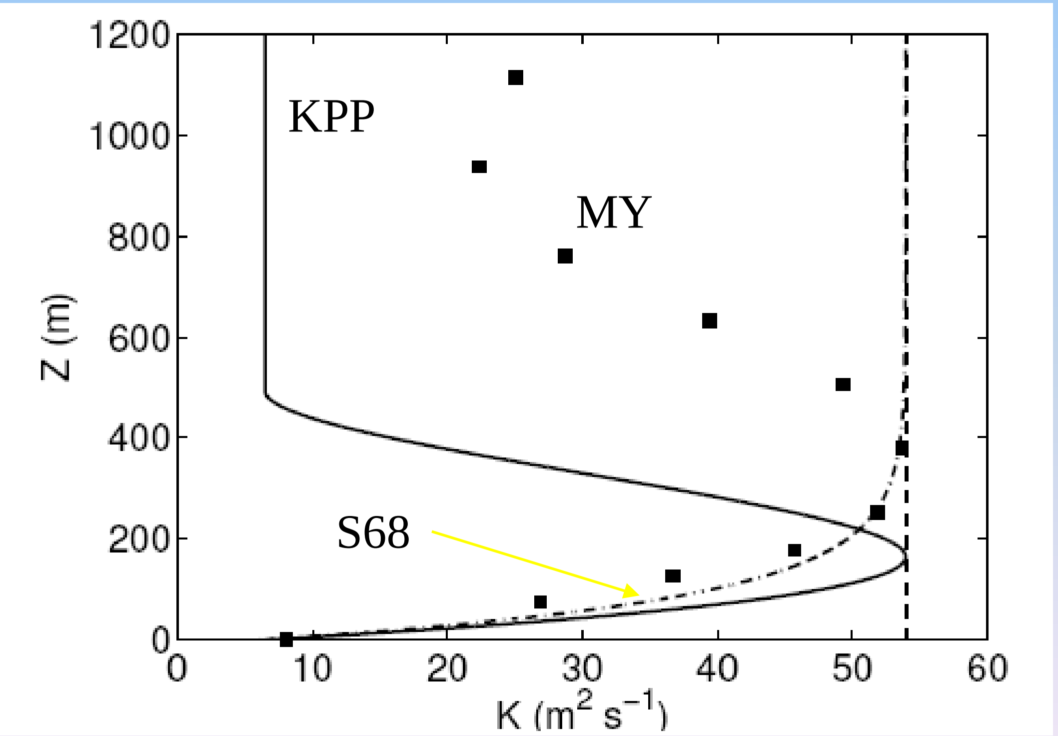
$$\left[\frac{\partial}{\partial t} + i\alpha U - \frac{1}{R_e}(D^2 - \alpha^2)\right]v + wV' + \frac{1}{R_o}u + \frac{1}{r_e}\left[-U\sin(\varepsilon)v + (2U\cos(\varepsilon) - V\sin(\varepsilon))u\right] =$$
$$-\left(uD_x + wD\right)v - \frac{1}{r_e}\left(\cos(\varepsilon)u - \sin(\varepsilon)v\right)u$$

$$\left[\frac{\partial}{\partial t} + i\alpha U - \frac{1}{R_e}(D^2 - \alpha^2)\right]w + Dp - R_i T_v = -\left(uD_x + wD\right)w$$

$$\left[\frac{\partial}{\partial t} + i\alpha U - \frac{1}{P_r R_e}(D^2 - \alpha^2)\right]T_v + w\bar{T}_v = -\left(uD_x + wD\right)T_v$$

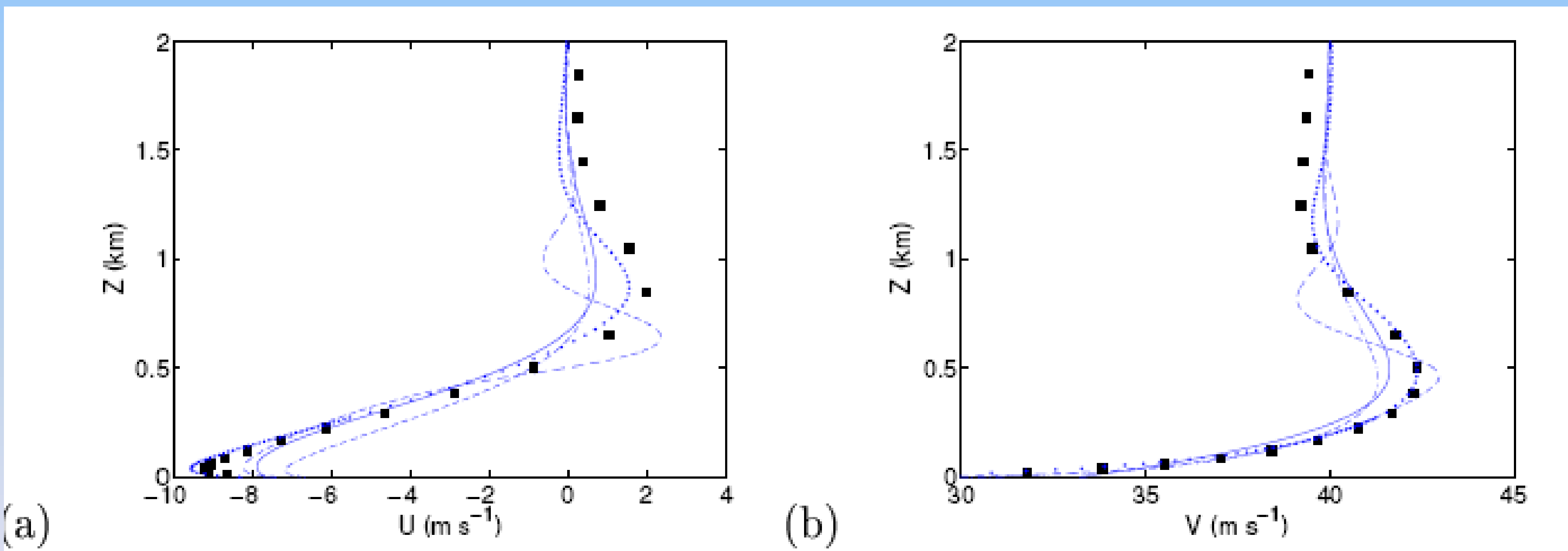
Turbulence Closure

- (Almost) Any eddy viscosity parameterization
- Have Implemented
  - K = *const*
  - K-Profile (ala Troen
  - Smith (1968)
  - Mellor-Yamada 2.0



R = 40 km

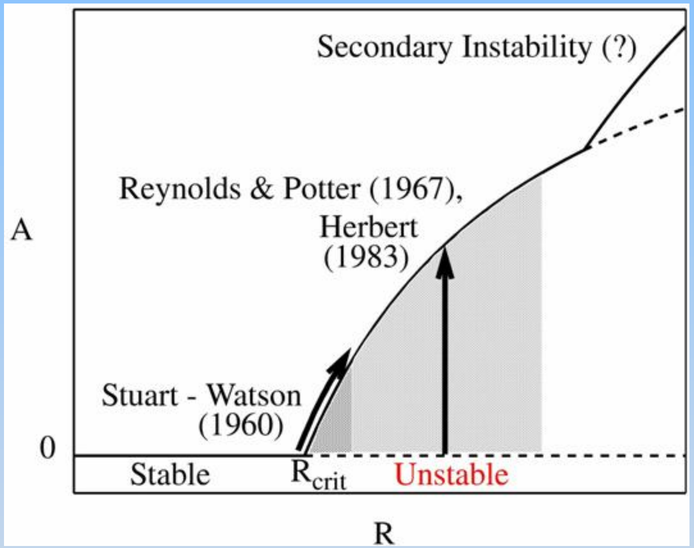
Radial Azimuthal



Black Dots are Nonlinear Kepert and Wang (2001) Numerical Model
Blue dotted line is Similarity Model Driven by Kepert & Wang K(z) (39 m/s)
→ *Similarity Model reproduces results of time-stepping numerical model!*

Nonlinear Roll Instability Model

Nonlinear Stability

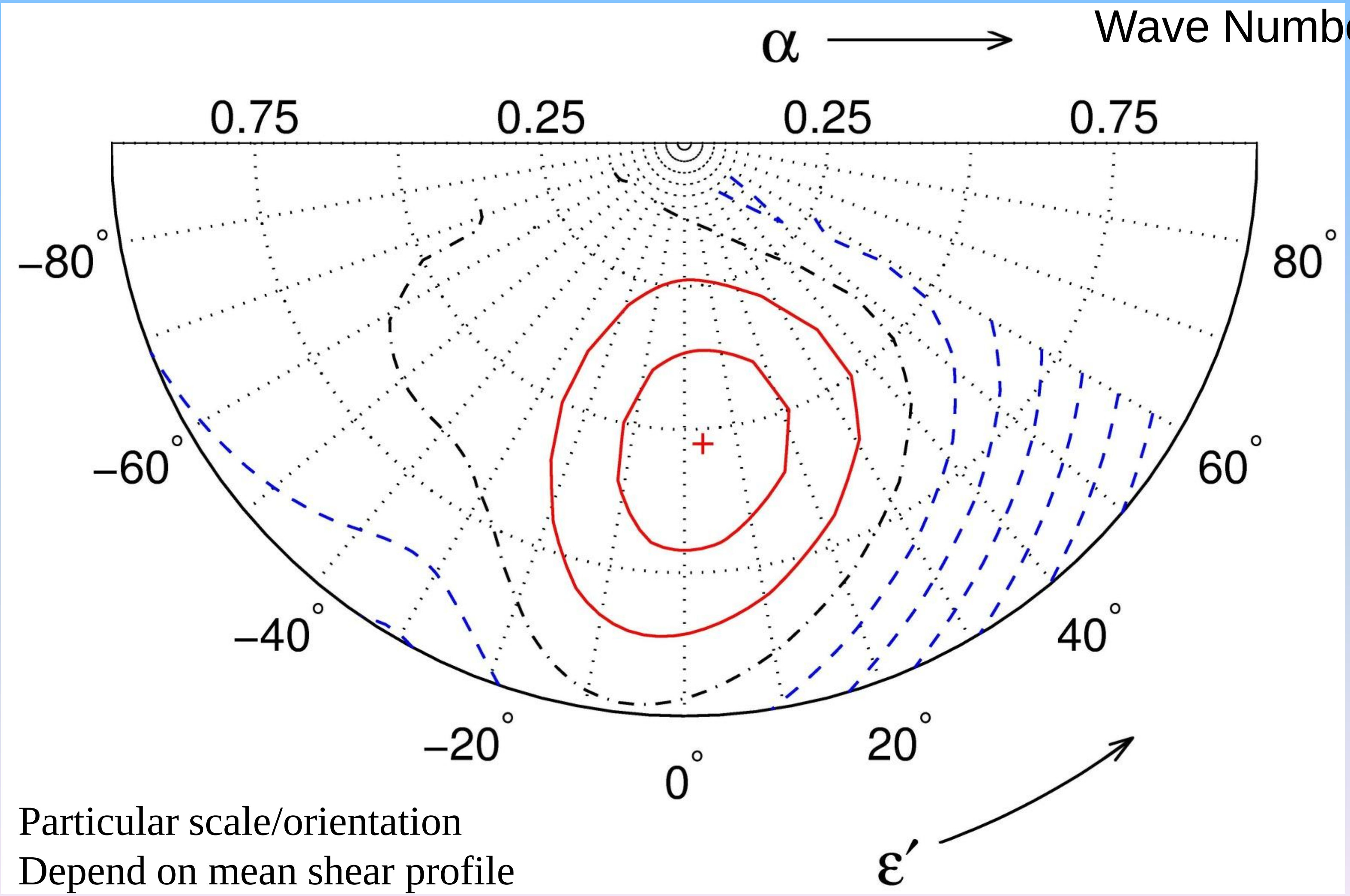


$$\lambda = a + i\omega = \frac{1}{A}\frac{dA}{dt} + i\frac{d\eta}{dt} = \lambda_0 + A^2\lambda_1 + A^4\lambda_2 + \cdots$$

$$\underset{\sim}{q} = 2real\left[\sum_{n=0}^{\infty} A^n e^{in(\alpha x - \omega t)} \sum_{m=0}^{\infty} A^{2m} \underset{\sim}{q}_{nm}(z)\right]$$

- “Stretch” eigenvalue, λ_0, in powers of nonlinear amplitude, *A(t)*.
- Expand eigenfunction, q_{10}, in harmonics of fundamental wavenumber, α.
- Find equilibrium solution ($dA/dt = 0$).

Growth Rate

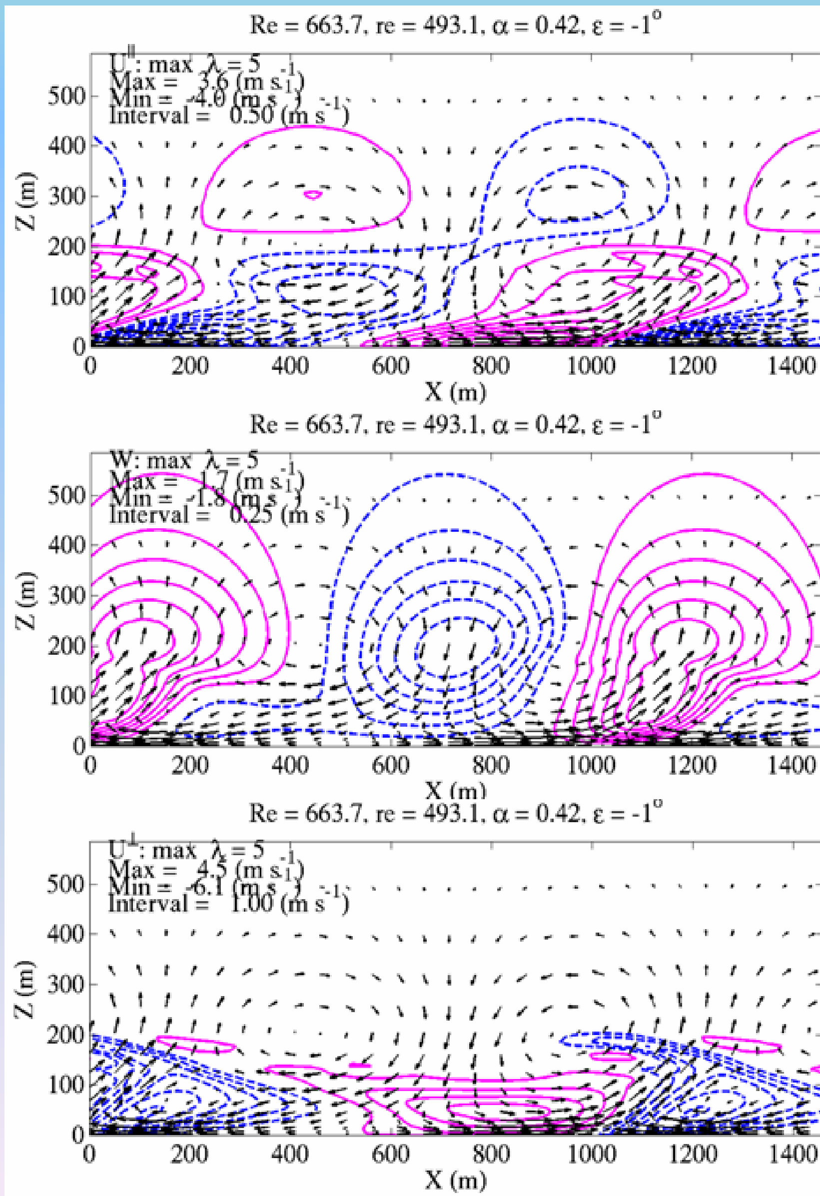


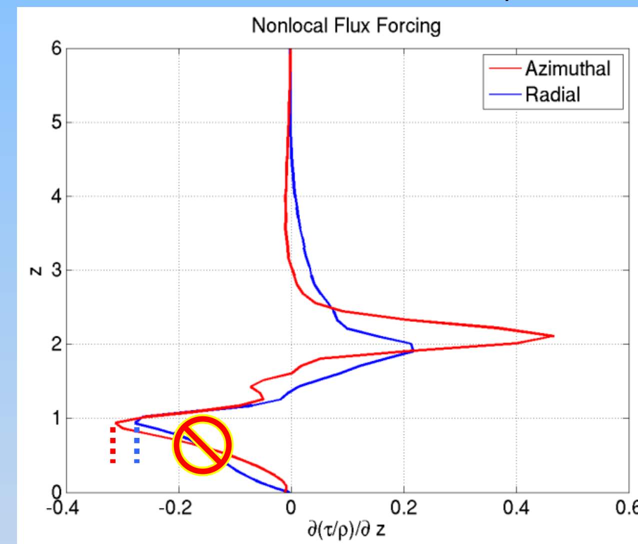
Particular scale/orientation
Depend on mean shear profile

Angle from Azimuthal Wind

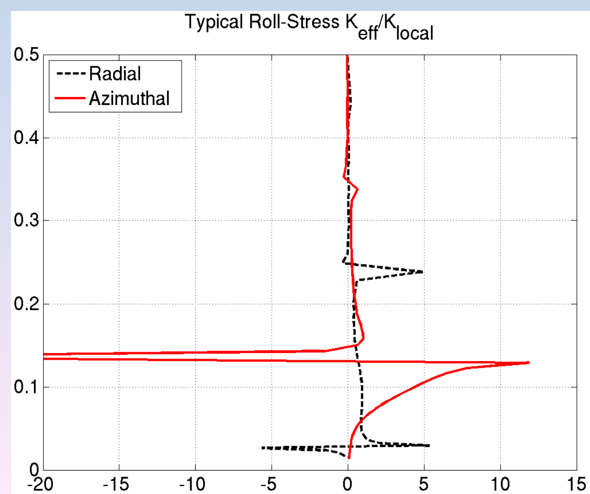
Improved version of Foster (2005) TCBL roll model & Parameterization development

$$\frac{\partial \tau}{\partial z} = \frac{\partial \tau_{local,existing\ PBL\ param.}}{\partial z} + \underbrace{\frac{\partial \tau_{non-local}}{\partial z}}$$



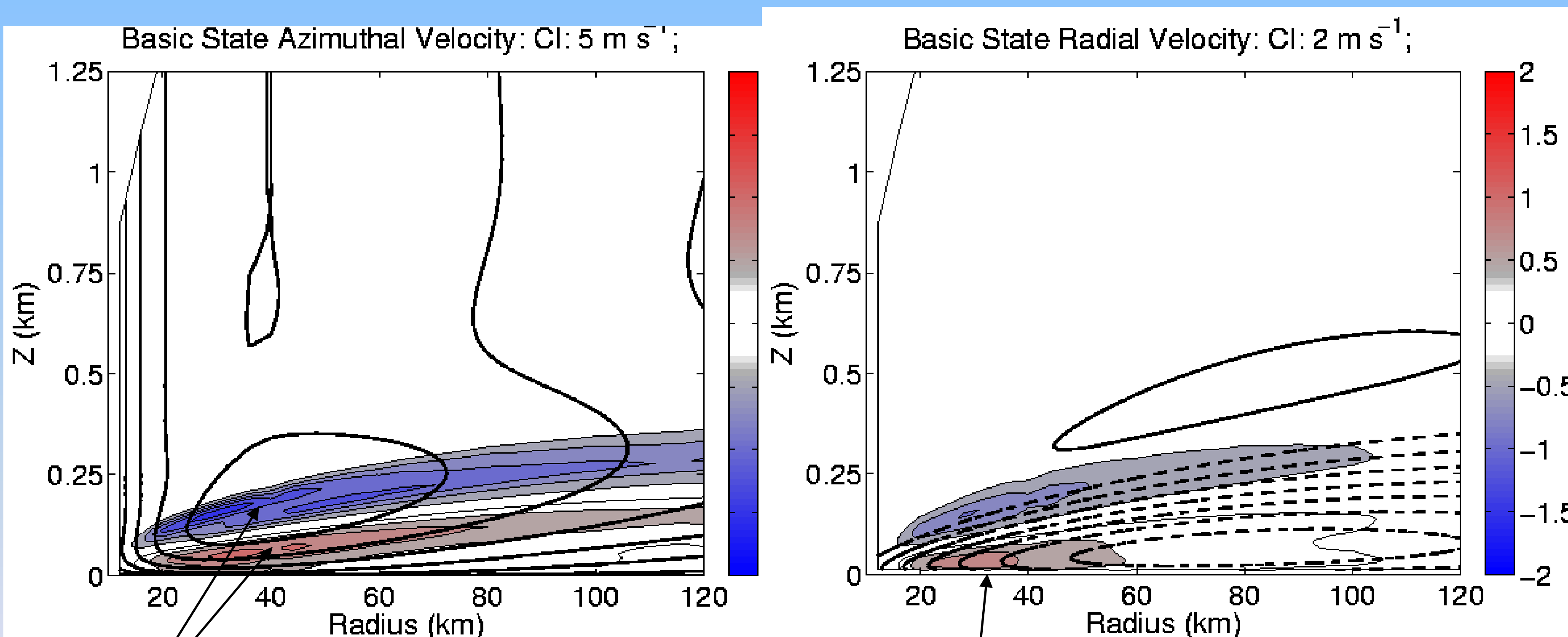


Mostly fixed near-surface roll fluxes



Cannot just “bump up” K

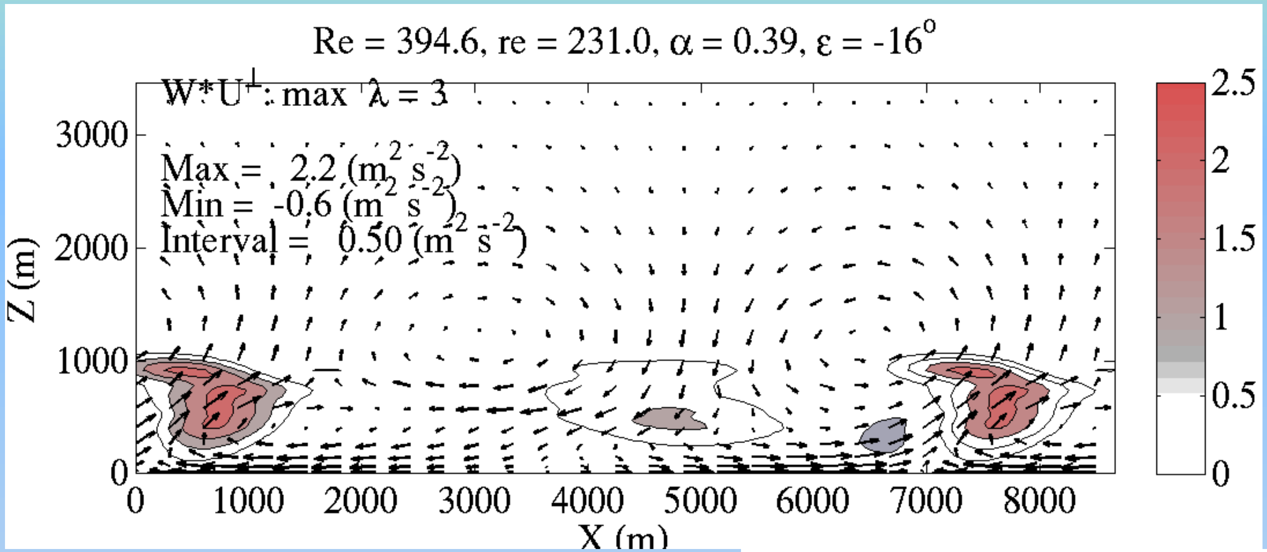
Roll-Induced Modification of the Mean Flow

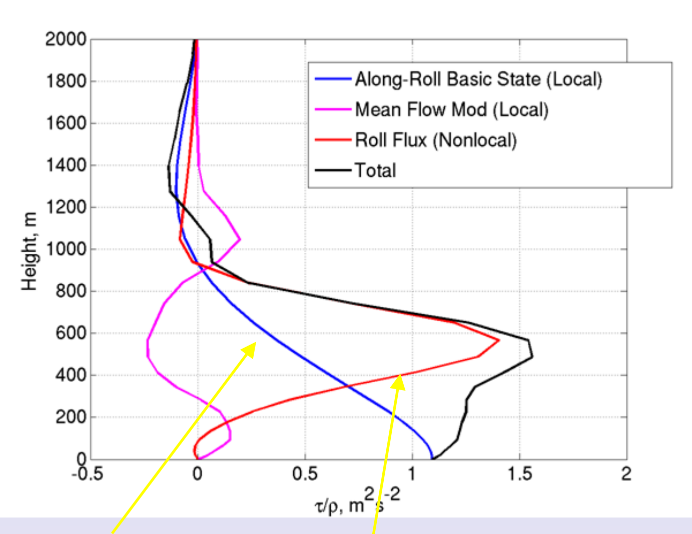


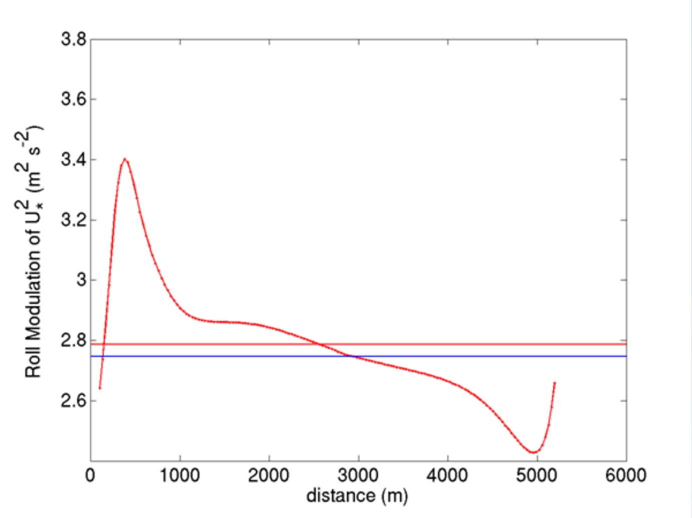
Transfer of Super-Gradient
Jet momentum towards surface

Weakened near-surface
inflow near and inside RMW

Non-local Momentum Flux



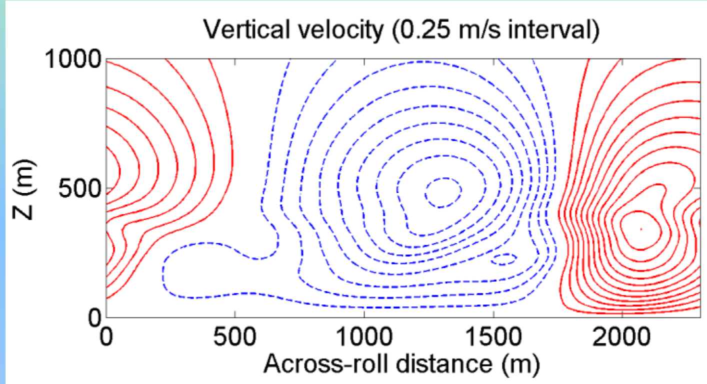


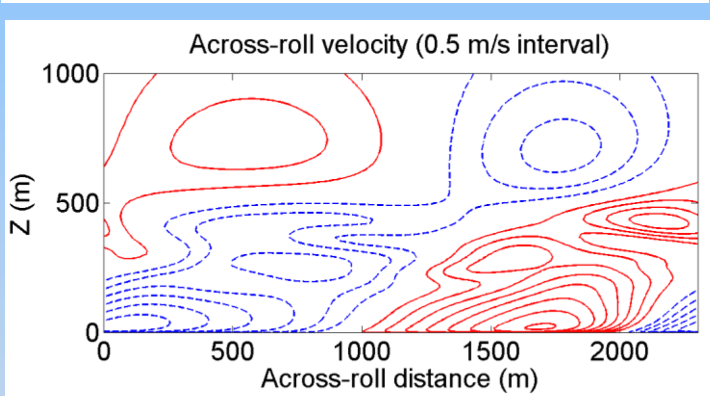


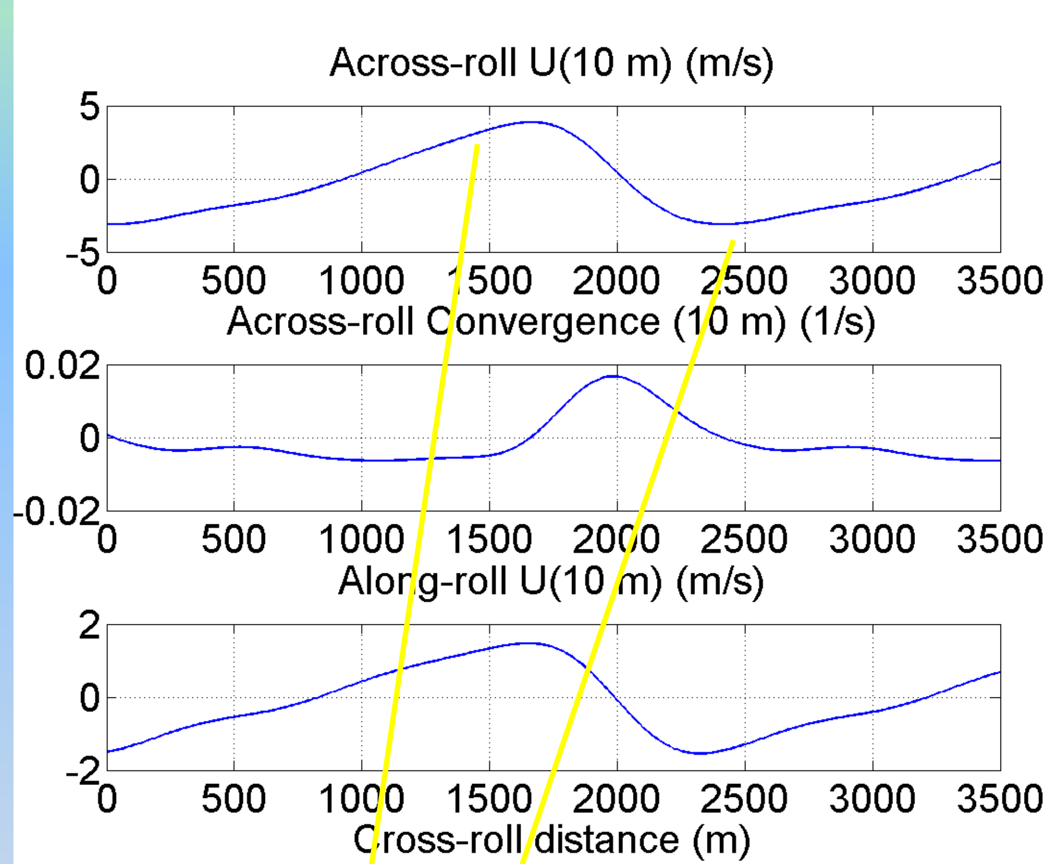
Local

Nonlocal

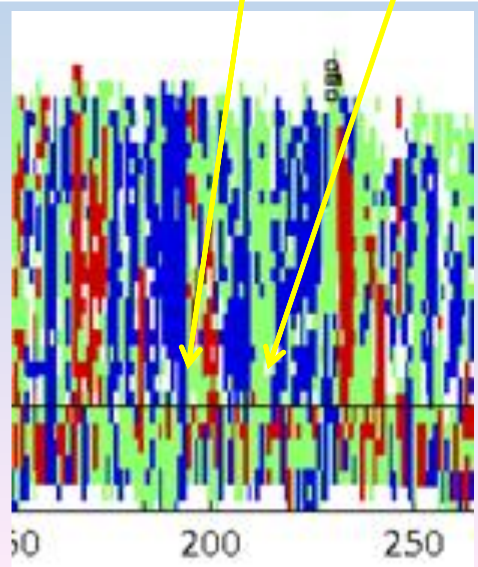
(These plots are from various calculations)







New OLE theory correctly connects PBL OLEs with the Surface Layer

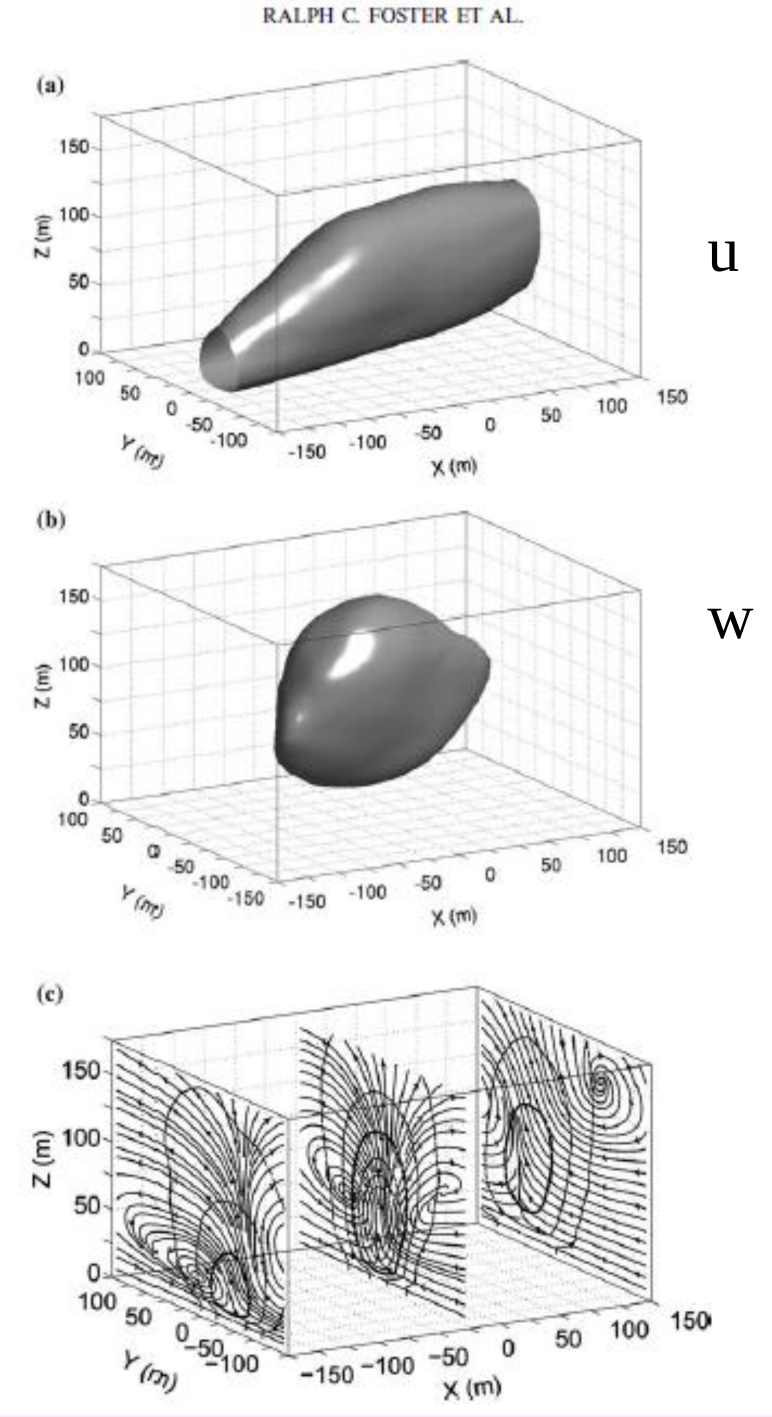


Are OLE dynamics and sea-surface Perturbation connected?

Plan

- Improve and develop Foster (2005) roll theory
  - Lower BC
  - Interface with similarity mean TCBL model
  - Improve scalar fluxes
- Improve mean TCBL similarity model
  - Re-do temperature implementation
  - Water vapor
- Analyze available SAR imagery (have large catalog including ATL and WPAC, from HW & ITOP)
  - Roll characteristics
  - Conditions when present & not present
- Collaborate with I. Ginis & K. Gao
  - Contrast methodologies
  - Work on paramterizations
  - Test implementations

Extra Slides



u

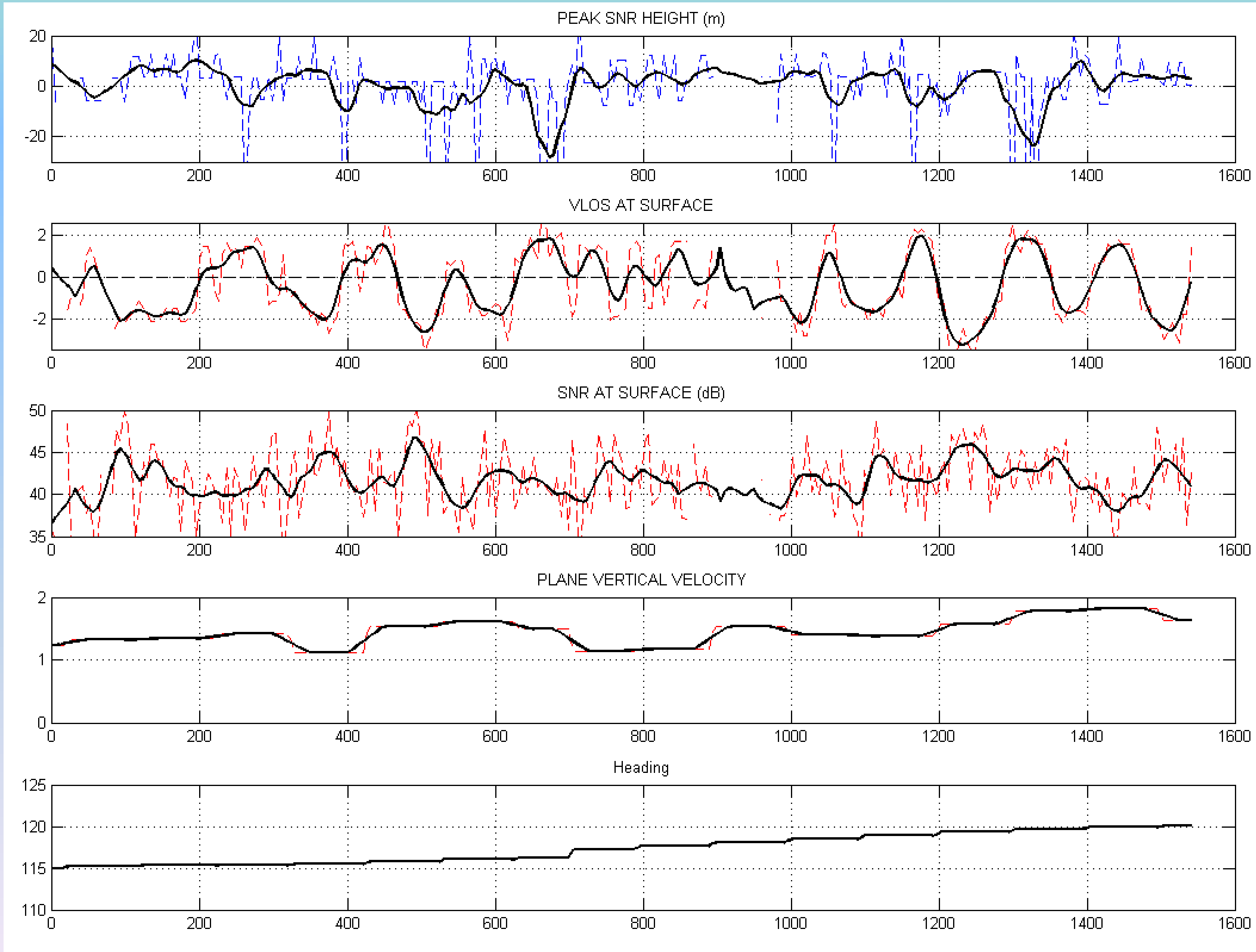
w

Conditionally-sampled ejection
Embedded in streak updraft

Boundary-Layer Meteorology (2006) **120**: 229–255

~10 – 12 m/s (into Hagupit)

Swell

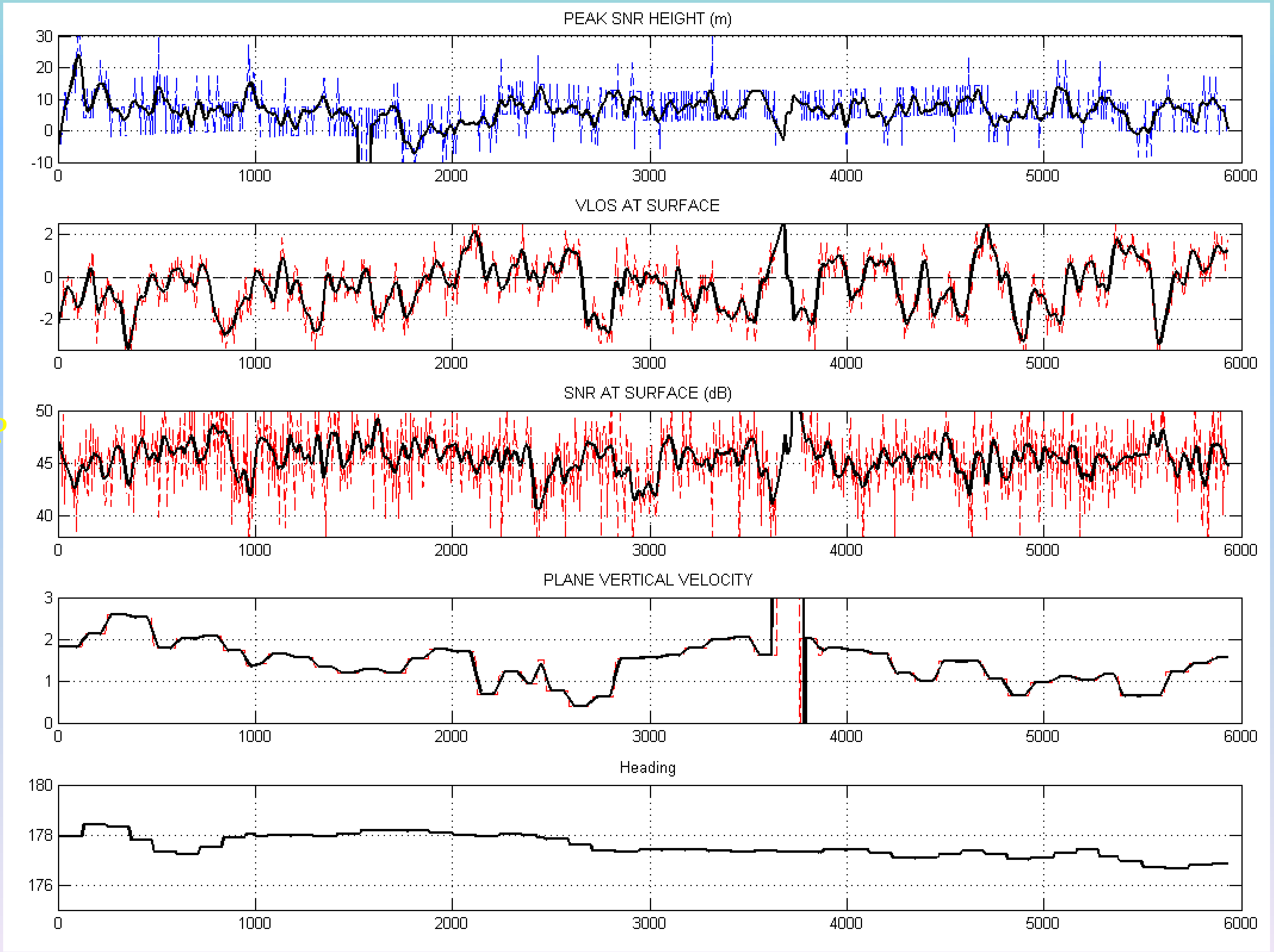


Horizontal scale in meters

~30 m/s

Hydrometeors?
Foam?
Spray?

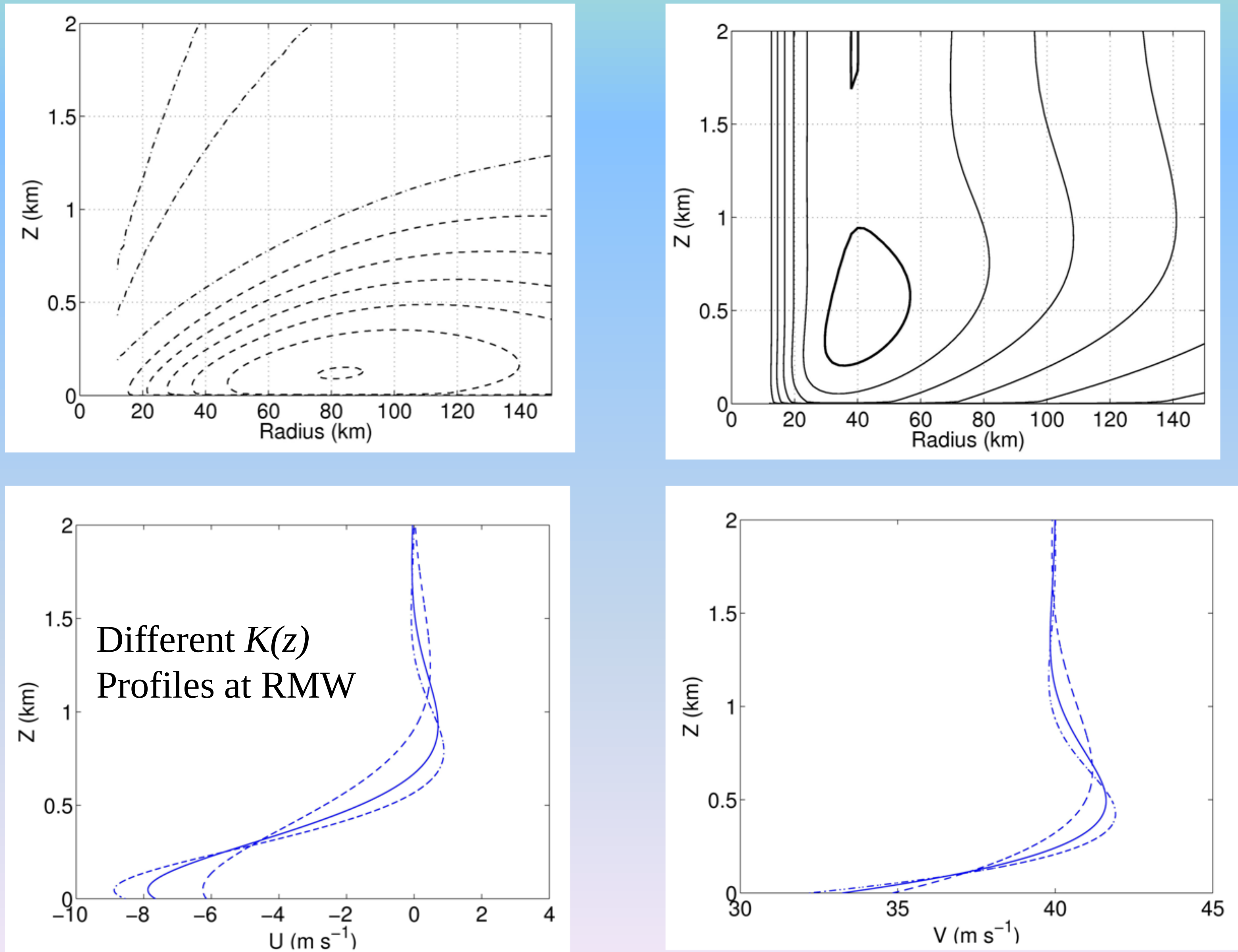
A/C vertical motion accounted for



Horizontal scale in meters

Radial

Azimuthal

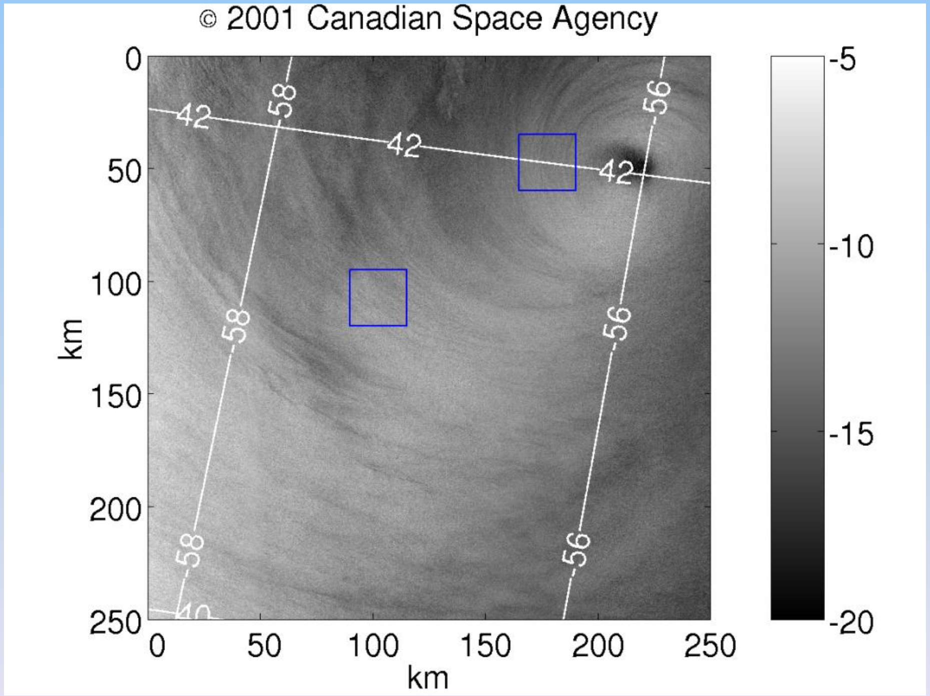


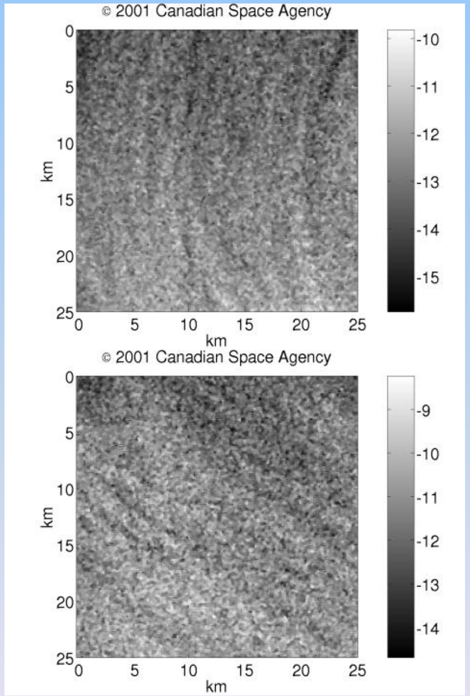
Z = 0 is “Top” of “Surface Layer”

PBL Rolls

- Organized Large Eddies
  - Roughly parallel to the mean PBL wind
  - Overturning circ. (v-w) spans the depth of the PBL
  - Much stronger along-roll (u) perturbation
  - Aspect ratio ~2.4 to 6 (wavelength/depth)
- Very common
- Basic characteristics agree with most unstable normal mode instability
  - Nonlinear strength ~5% to ~20% of mean flow
  - Can be quite large in hurricane BLs
- Lateral phase velocity

Synthetic Aperture Radar





Non-Dimensionalize and Scale Full Perturbation Equations in Cylindrical Coordinates

Example: Radial Momentum Equation

$$\frac{V_g^2}{\delta}\left\{\frac{\partial u'}{\partial t}+U\frac{\partial u'}{\partial r}+V\frac{\partial u'}{r_e\partial\theta}+w'\frac{\partial U}{\partial z}\right\}+\frac{V_g^2}{\delta}\frac{1}{r_e}\ -2Vv'\ +fV_g\ -v'\ =$$

$$\frac{\rho_0 V_g^2}{\rho_0\delta}\ -\frac{\partial p'}{\partial r}\ +\frac{KV_g}{\delta^2}\left\{\frac{\partial^2 u'}{\partial r^2}+\frac{\partial^2 u'}{r_e^{\,2}\partial\theta^2}+\frac{\partial^2 u'}{\partial z^2}\right\}+\frac{KV_g}{\delta^2}\frac{1}{r_e}\left\{\frac{\partial u'}{\partial r}-2\frac{\partial v'}{r_e\partial\theta}\right\}+\frac{KV_g}{\delta^2}\frac{1}{r_e^2}\ -u'\ -$$

$$\frac{V_g^2}{\delta}\left\{u'\frac{\partial u'}{\partial r}+v'\frac{\partial u'}{r_e\partial\theta}+w'\frac{\partial u'}{\partial z}\right\}+\frac{V_g^2}{\delta}\frac{1}{r_e}\ \left\{v'^2\ \right\}$$

$$\frac{\partial u'}{\partial t}+U\frac{\partial u'}{\partial r}+V\frac{\partial u'}{r_e\partial\theta}+w'\frac{\partial U}{\partial z}-\frac{1}{r_e}2Vv'-\frac{1}{R_o}v'=$$

$$-\frac{\partial p'}{\partial r}+\frac{1}{R_e}\left[\frac{\partial^2 u'}{\partial r^2}+\frac{\partial^2 u'}{r_e^{\,2}\partial\theta^2}+\frac{\partial^2 u'}{\partial z^2}\right]+\frac{1}{R_e}\frac{1}{r_e}\left[\frac{\partial u'}{\partial r}-2\frac{\partial v'}{r_e\partial\theta}\right]-\frac{1}{R_e}\frac{1}{r_e^2}\ u'\ -$$

$$-\left[u'\frac{\partial u'}{\partial r}+v'\frac{\partial u'}{r_e\partial\theta}+w'\frac{\partial u'}{\partial z}\right]-\frac{1}{r_e}\left[v'^2\right]$$

Nondimensionalized Equation: Ready to Scale

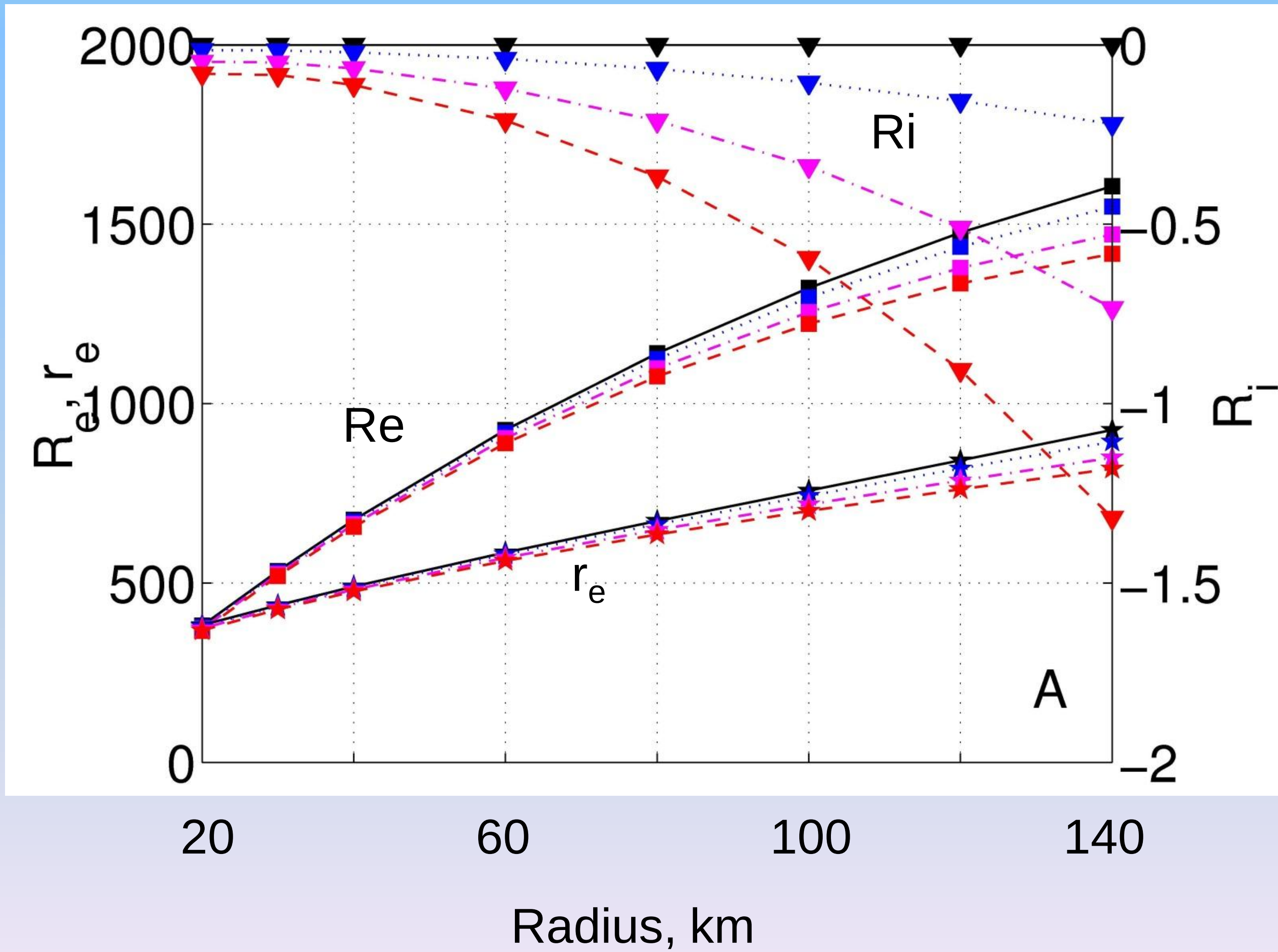
Perturbation E^{qn} Scaling

- Scaling for separable equations:
  - Locally Cartesian & Parallel
  - plane wave solutions: $\alpha^2 = (\alpha_x{}^2 + \alpha_y{}^2)$
- $r = r_e$ (constant)
- $dr = dx$
- $r_e d\theta = dy$
- Keep R_e^{-1} and r_e^{-1} terms
- Drop terms of $(R_e r_e)^{-1}$ or smaller
- Keep r_e^{-1} terms only if multiply mean flow term
- Squire's transformation ($\varepsilon + 90^o$)

Rolls

- Form perturbation equations ($U_{tot} = U + u, \ldots$)
- Linearize, look for "fastest growing normal mode"
  - Do characteristics agree with observations?
- Perform nonlinear analysis of fastest growing mode
  - Seek equilibrium solutions
    - Calculate finite magnitude
    - Agree with Observations?

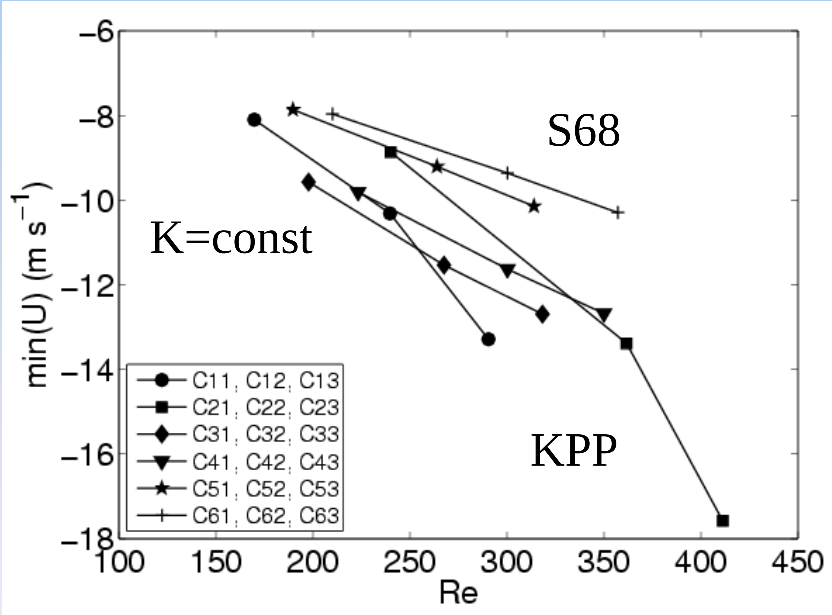
“Generic Hurricane” RMW = 40 km, Vmax = 40 m/s, B = 1.3



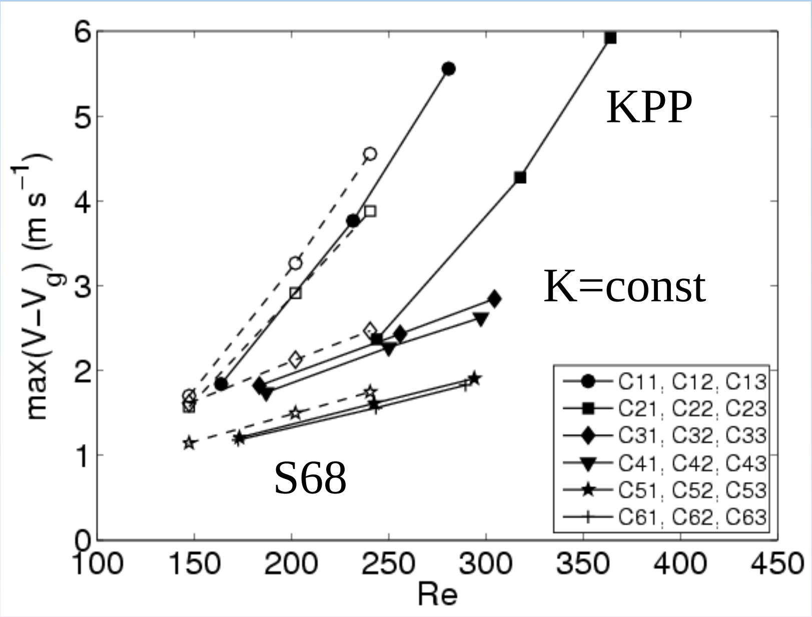
ΔT_v = 0, -1, -3, -5

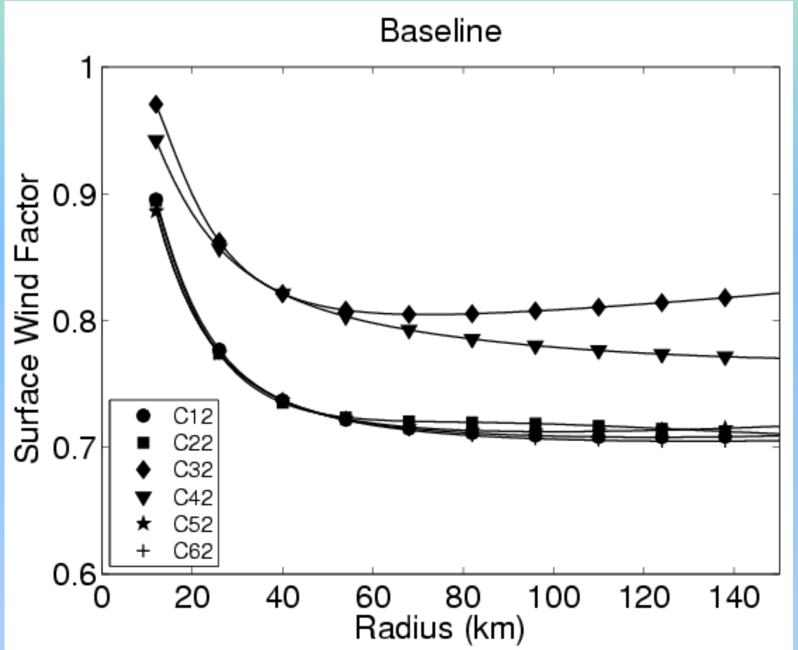
Re is the Vortex Boundary Layer Flow Parameter
(For any particular turbulence closure)

Max Inflow

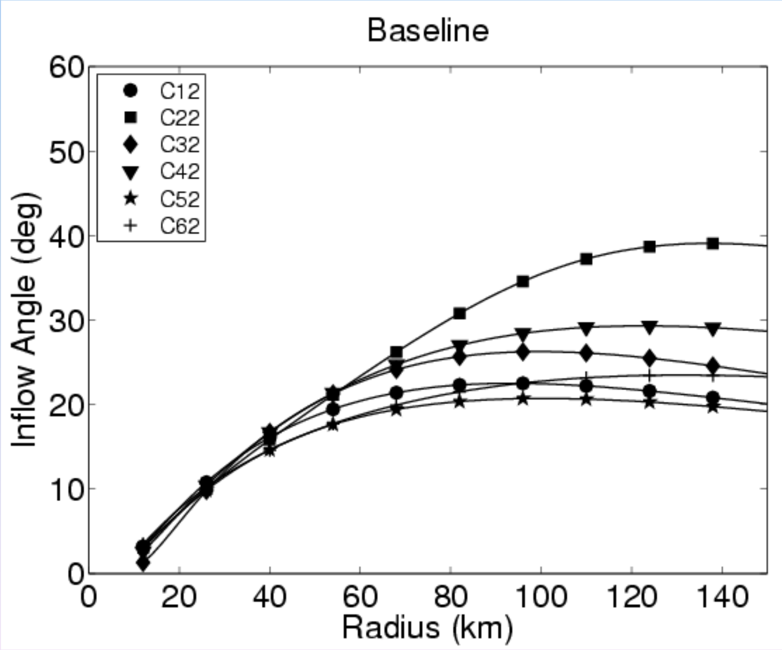


Max Super-Gradient Jet



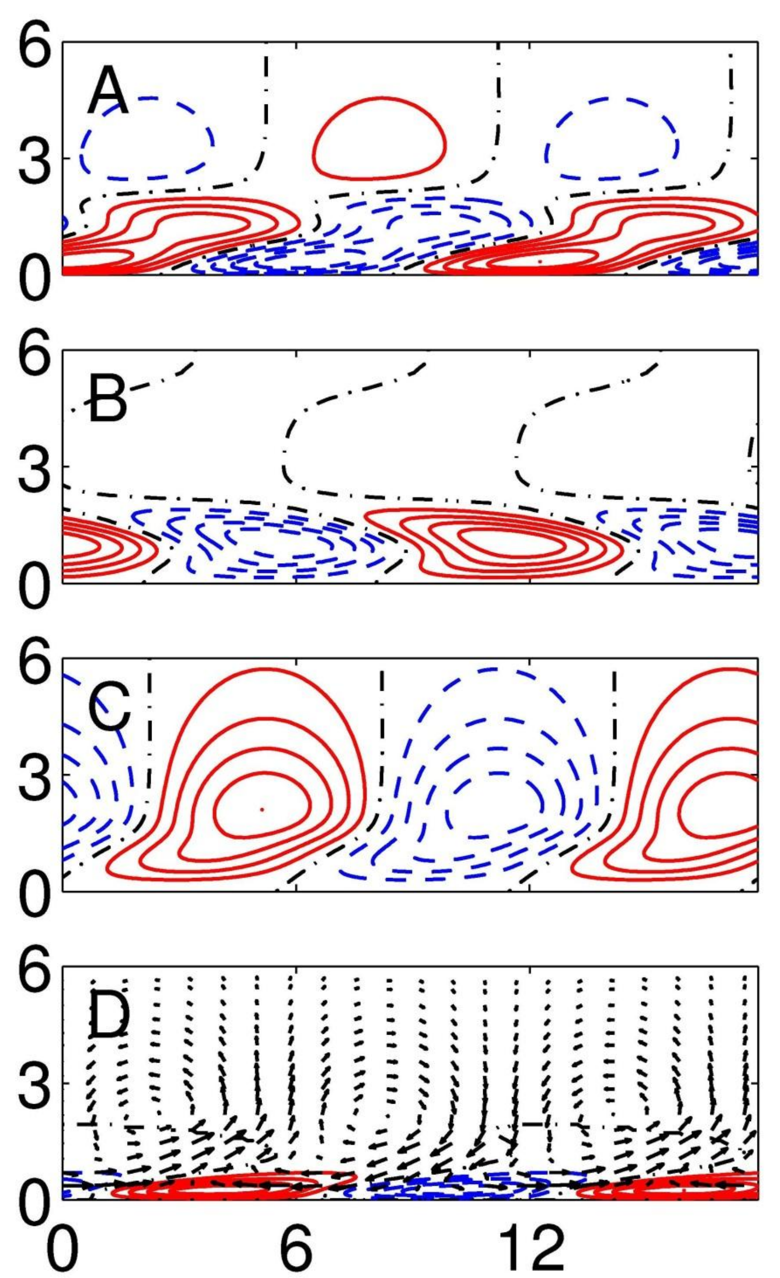


For closures that match C_D, C_D controls Usfc/Vg



But, closure controls inflow angle

Typical
Normal Mode



Across-Roll

Along-Roll

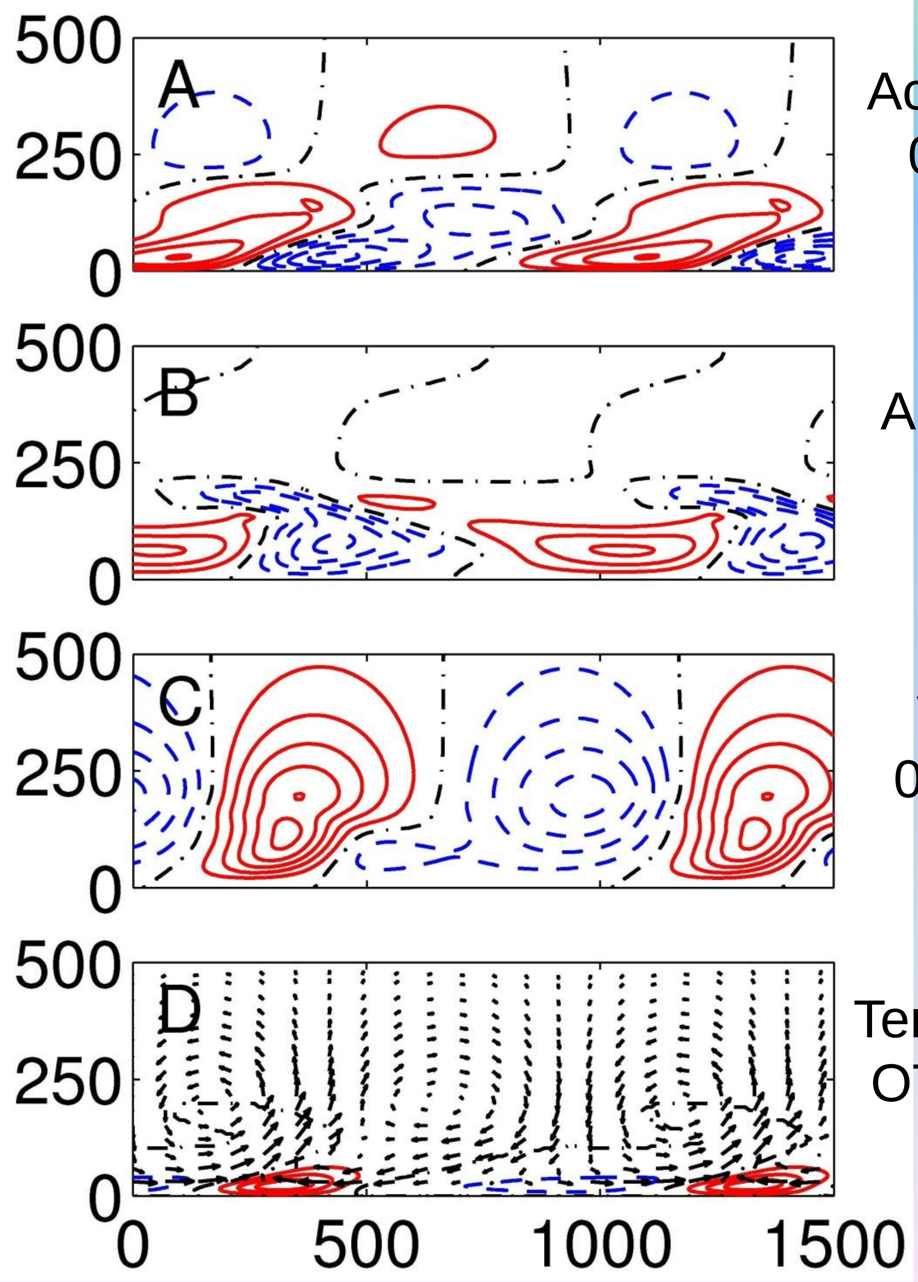
Vertical

Temperature
OT Vectors

Old roll model
to be superceeded

Narrow, strong updrafts

Broad, weaker downdrafts



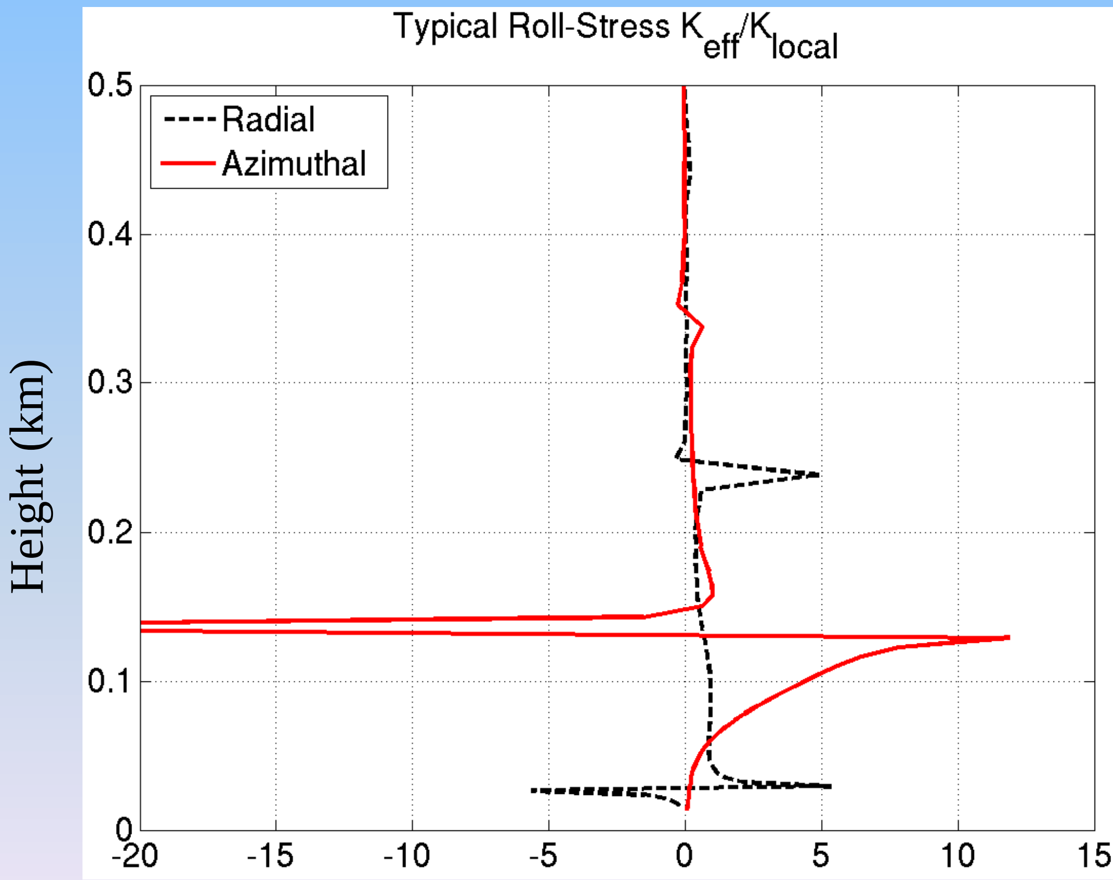
Across-Roll
0.5 ms^{-1}

Along-Roll
1 ms^{-1}

Vertical
0.25 ms^{-1}

Temperature
OT Vectors
0.2 K

Nonlocal Roll-flux doesn't conform with standard gradient-flux modeling

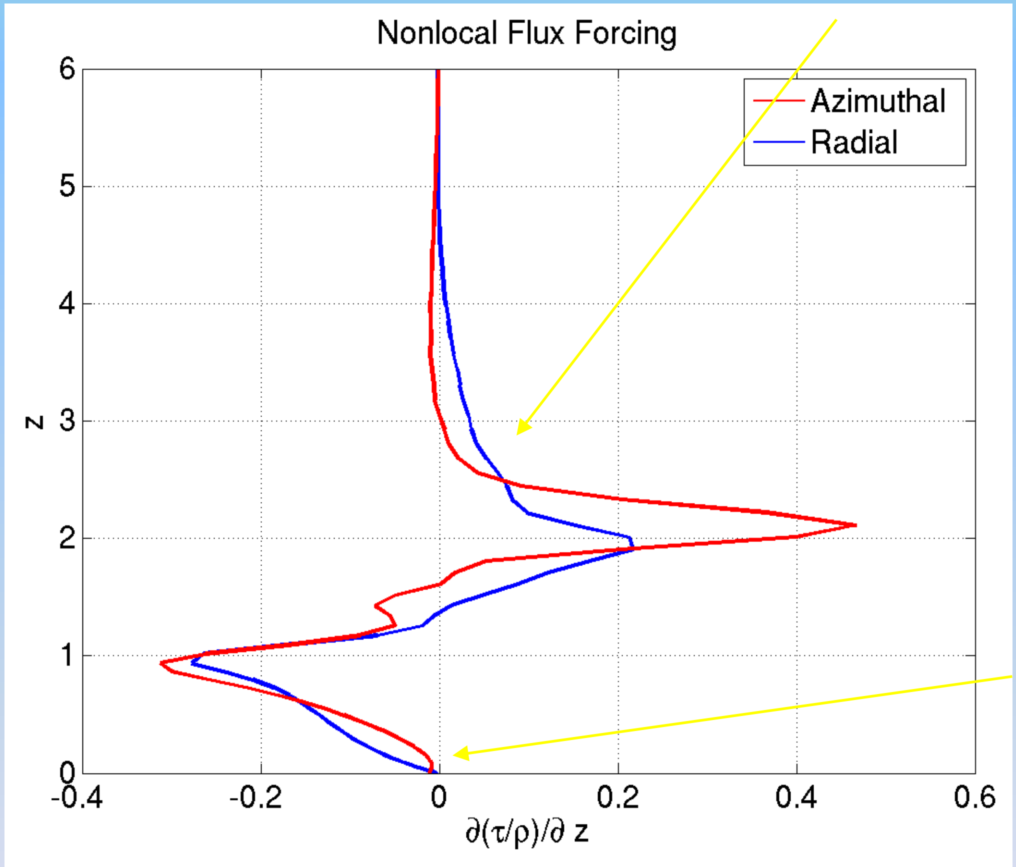


"K_{eff}" (needed to generate nonlocal flux)/K_{local}

- Negative *K* is unphysical
- "K_{eff}" different for Radial and Azimuthal flow
- Requires very large values and rapid changes

Based on Foster (2005)

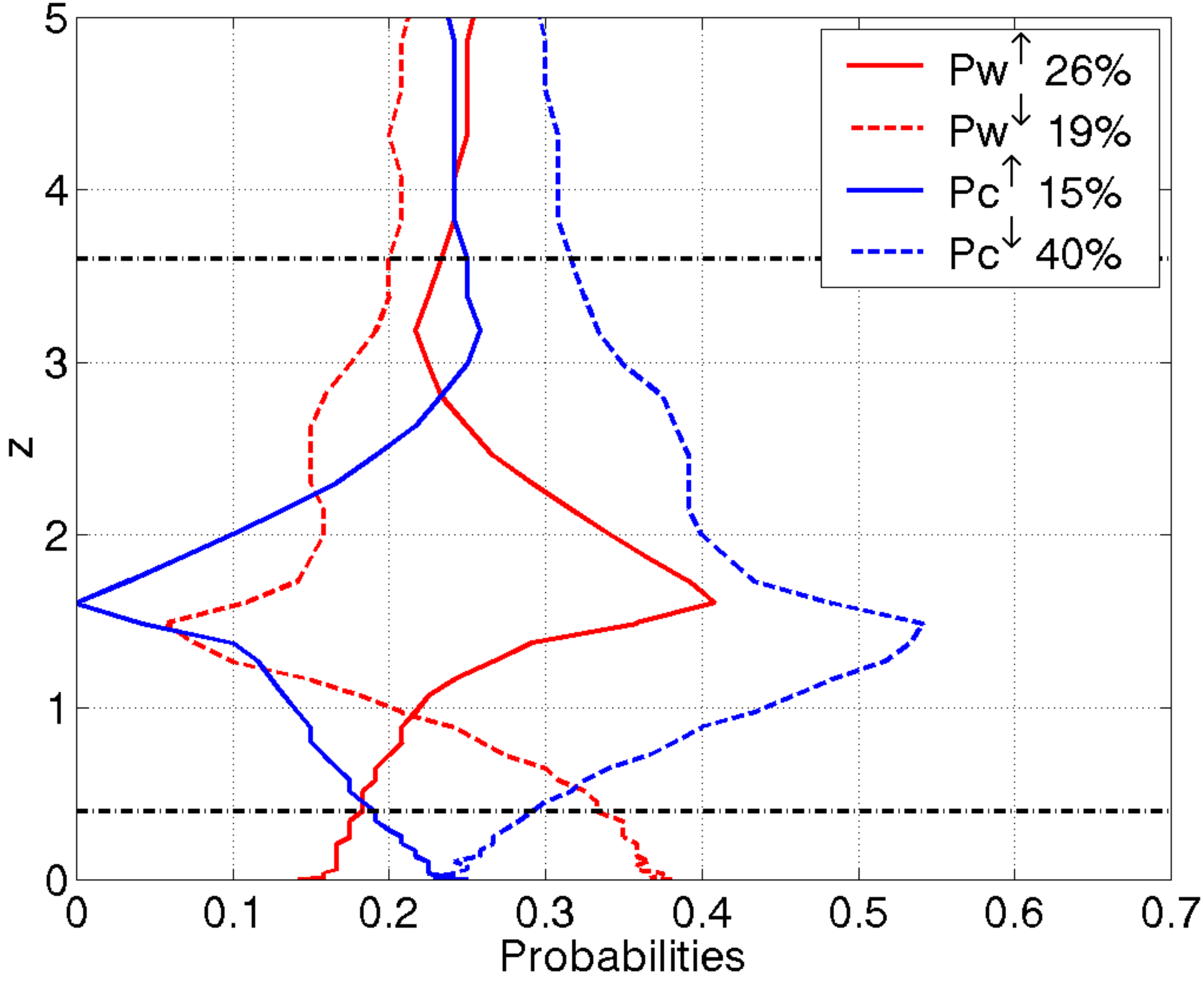
$$\frac{\partial \tau}{\partial z} = \frac{\partial \tau_{local,existing\ PBL\ param.}}{\partial z} + \underbrace{\frac{\partial \tau_{non-local}}{\partial z}}$$



This calculation omits surface effects (Mostly Fixed in recent weeks)

• Conceptual model for non-local roll flux parameterization. Appears to be simple to parameterize & *incorporate into* ***existing*** *PBL parameterizations*.

• Numerical models are approaching roll-scale resolution; simple averaging (like this) may not suffice. *However, neither will existing parameterizations*.



Pw↑ 26%
Pw↓ 19%
Pc↑ 15%
Pc↓ 40%
z
Probabilities
0
1
2
3
4
5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Streaks

- Transient, near-surface features of nearly all strongly-sheared BL flows
- Simplest theory:
  - Explosive growth of ***non-modal*** perturbations
  - Possible even when all normal modes are stable
  - Can form in the presence of unstable normal modes
  - Analogous to adjoint forecast sensitivity analysis
- Role in maintenance of surface stress

Boundary-Layer Meteorology (2006) **120**: 229–255

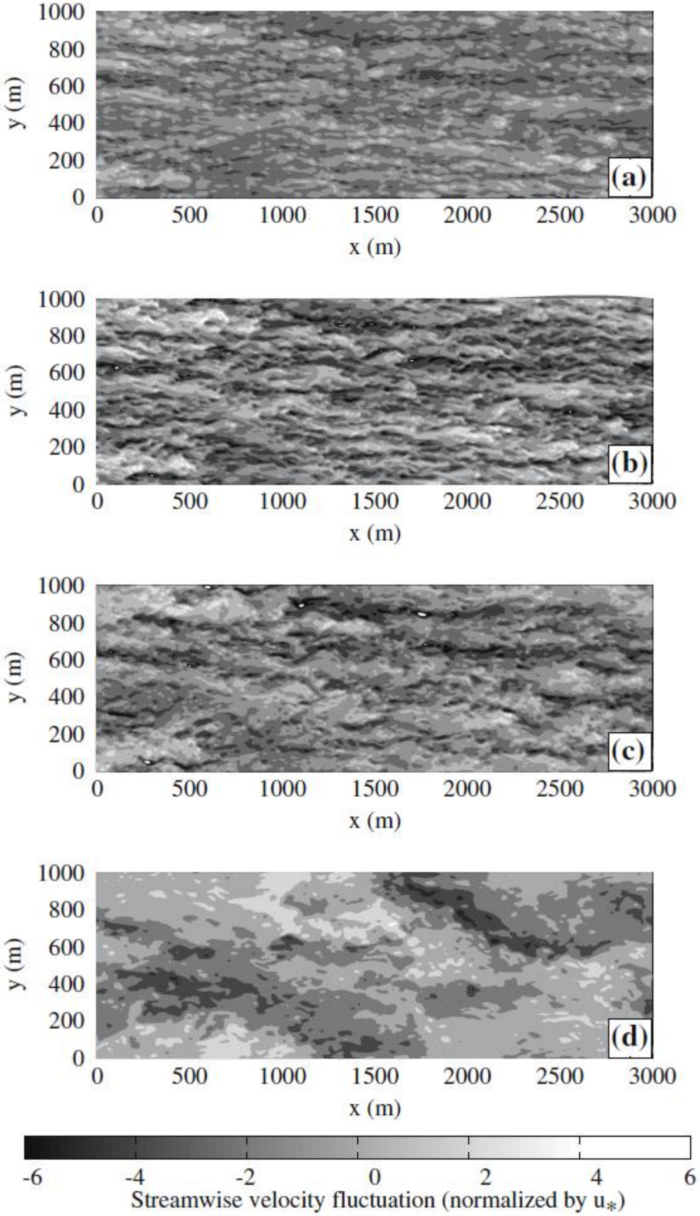


Figure 2. Snapshot of horizontal cross-sections (x–y) of the turbulent longitudinal (or streamwise) velocity u at $z = 9$ m (a), $z = 28$ m (b), $z = 47$ m (c) and $z = 153$ m (d). Negative u appears in dark shading and positive u in light shading.

Note initial “lean into shear”
and rotation as it evolves

ORIGIN OF NEAR-SURFACE STREAKS 251

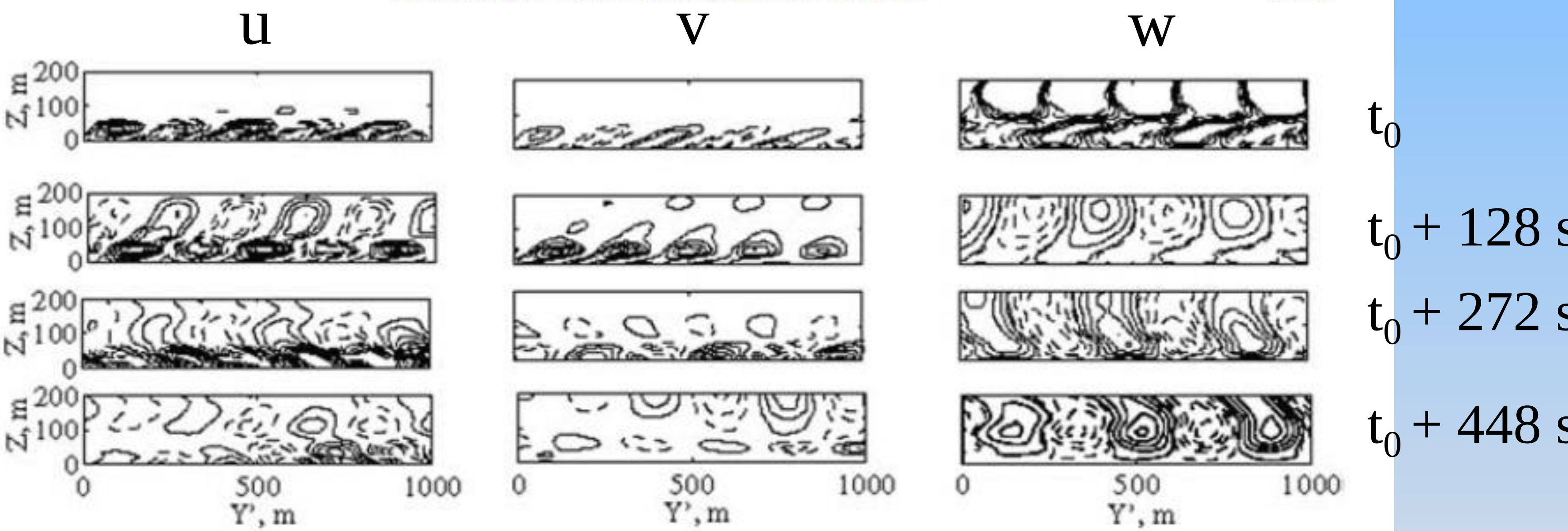


Figure 2. Contours of spatially-filtered u' v' and w (first, second and third columns respectively) in a vertical plane perpendicular to the 380 m wavelength streaks at times increasing downwards, 2784, 2912, 3056 and 3232 s respectively. Contour interval is 0.2 m s^{-1}. The zero contour is omitted. The maximum contour shown is ±1 m s^{-1}.

Boundary-Layer Meteorology **108:** 247–256, 2003

Overturning Streamfunction

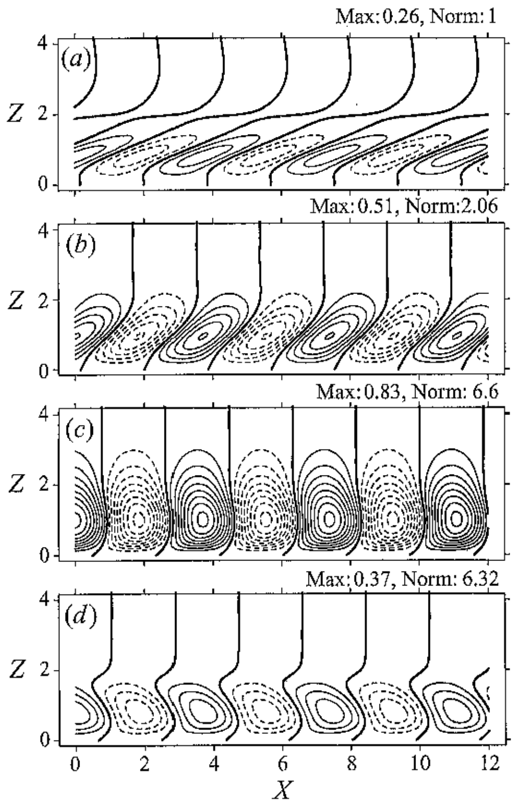


FIGURE 7. Contours of the streamfunction in the plane normal to the roll axis for the $\tau = 15$ optimal perturbation at (*a*) $t = 0$, (*b*) $t = 5$, (*c*) $t = 15$, and (*d*) $t = 25$. The wavenumber is 1.700 and the orientation angle is 29.5° from the surface isobars. The contour interval is 0.1 for all times. Solid contours are positive values and dashed contours are negative. The heavy solid lines mark the zero contour.

Down-roll velocity

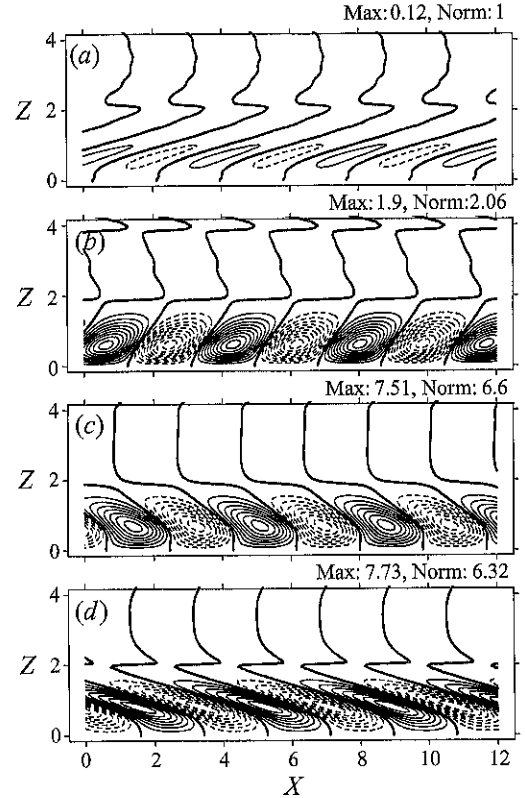


FIGURE 8. Contours of the downstream velocity in the plane normal to the roll axis for the $\tau = 15$ optimal perturbation at (*a*) $t = 0$, (*b*) $t = 5$, (*c*) $t = 15$, and (*d*) $t = 25$. The wavenumber if 1.700 and the orientation angle is 29.5° from the surface isobars. The contour interval is (*a*) 0.1, (*b*) 0.2, (*c*) and (*d*) 1. Solid contours are positive values and dashed contours are negative. The heavy solid lines mark the zero contour.

J. Fluid Mech. (1997), *vol.* 333, *pp.* 97–123

This is a roll

Explosive growth comes from non-orthogonal normal modes near branching of discrete spectrum (dashed-contours, logarithmic intervals)

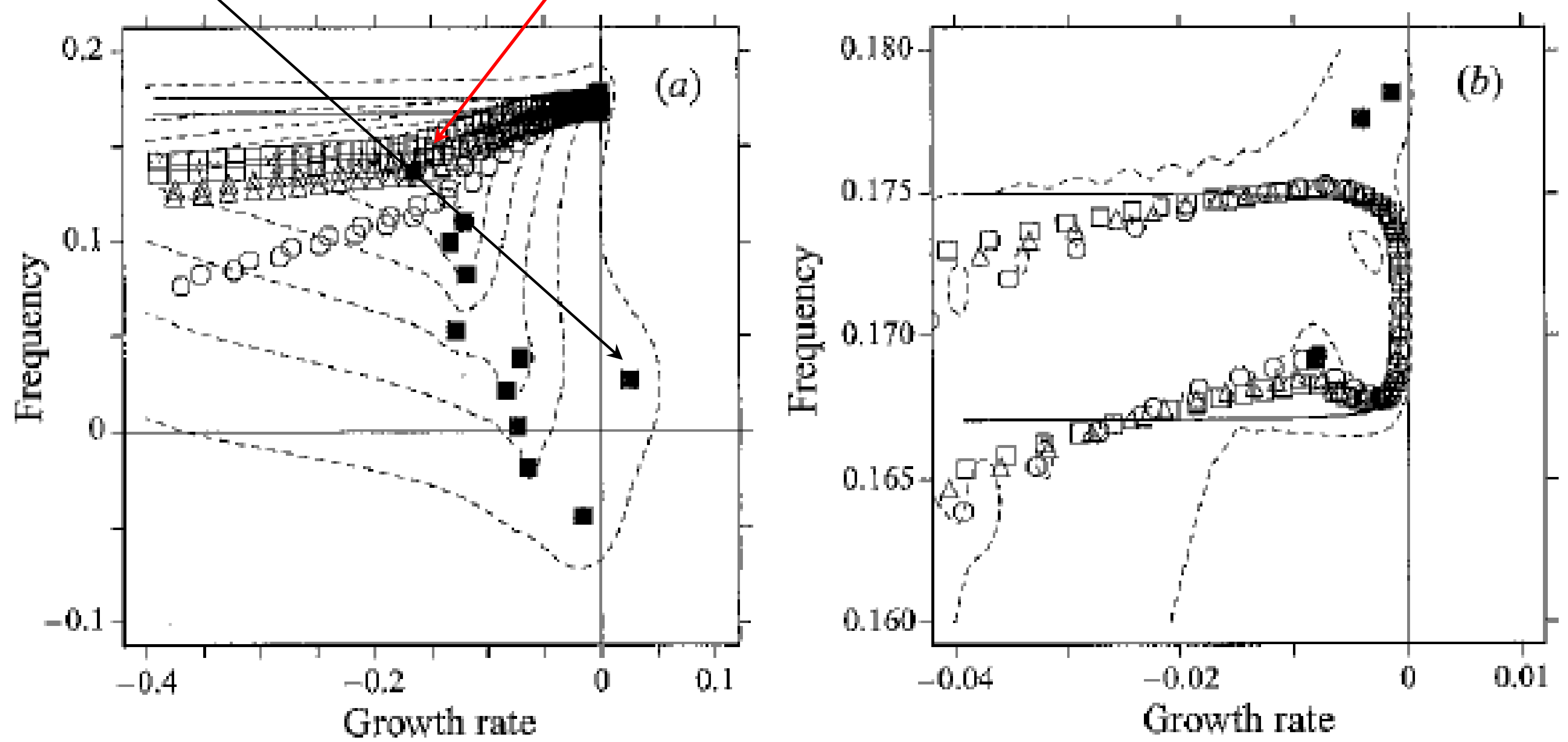


FIGURE 2. (*a*) Effect of resolution on the eigenvalues for the conditions $\alpha = 0.5$, $\epsilon = 20°$, $Re = 500$, neutrally stratified, barotropic and no tangential Coriolis force for $N_{poly} = 200$ (squares), 120 (triangles) and 60 (circles). Filled symbols are discrete normal modes and hollow symbols are discrete representations of elements on the continuous spectrum. (*b*) Expanded view near the continuous spectrum. In both (*a*) and (*b*) the predicted behaviour of the continuous spectrum is plotted as solid lines. Contours of the ϵ-pseudospectrum are plotted as dashed lines. In (*a*) the contours change by factors of 10 from 10^{-7} (inner) to 10^{-2} (outer). In (*b*) only the 10^{-3} and 10^{-4} contours are shown.

Short duration (transient):
small scale, close to sfc wind

Longer duration:
Large scale, becomes rolls

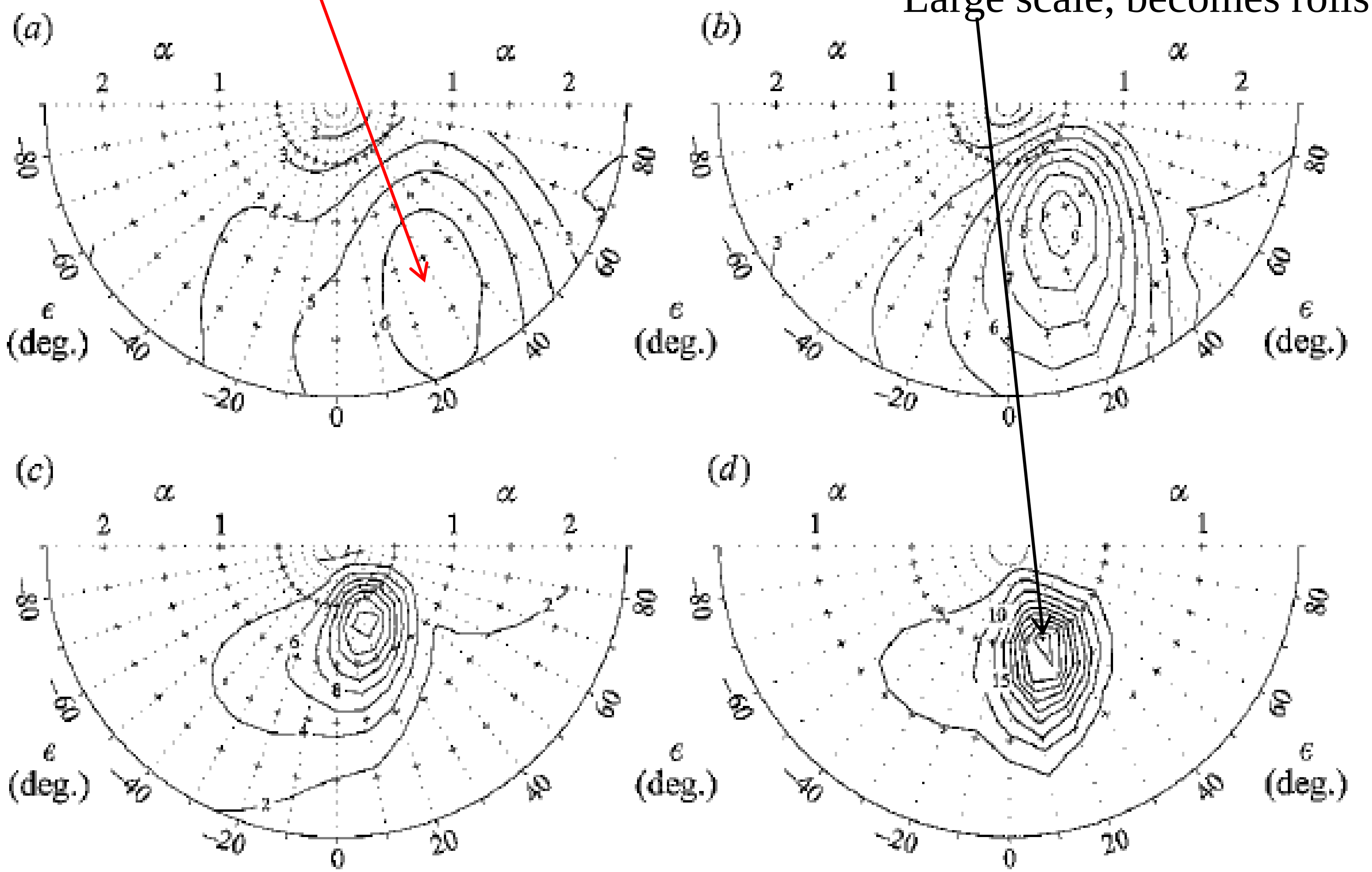
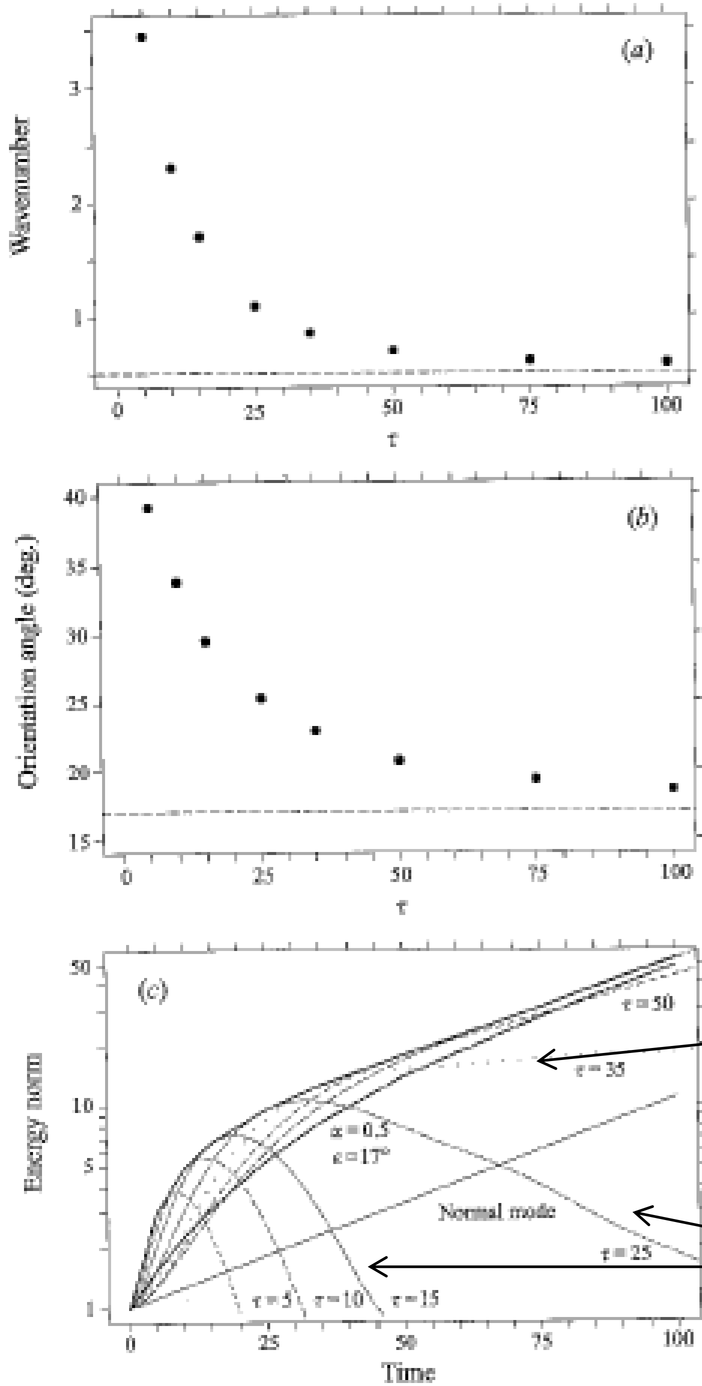


FIGURE 5. Contour plots of the maximum possible energy norm for $Re = 500$ as a function of wavenumber, α, and orientation angle, ϵ for time intervals (a) $\tau = 15$; (b) $\tau = 25$; (c) $\tau = 50$; (d) $\tau = 100$. The contour interval is 1 in (a) and (b); 2 in (c) and 5 in (d).

Most dangerous perturbation

Most dangerous perturbation

Energy Gain

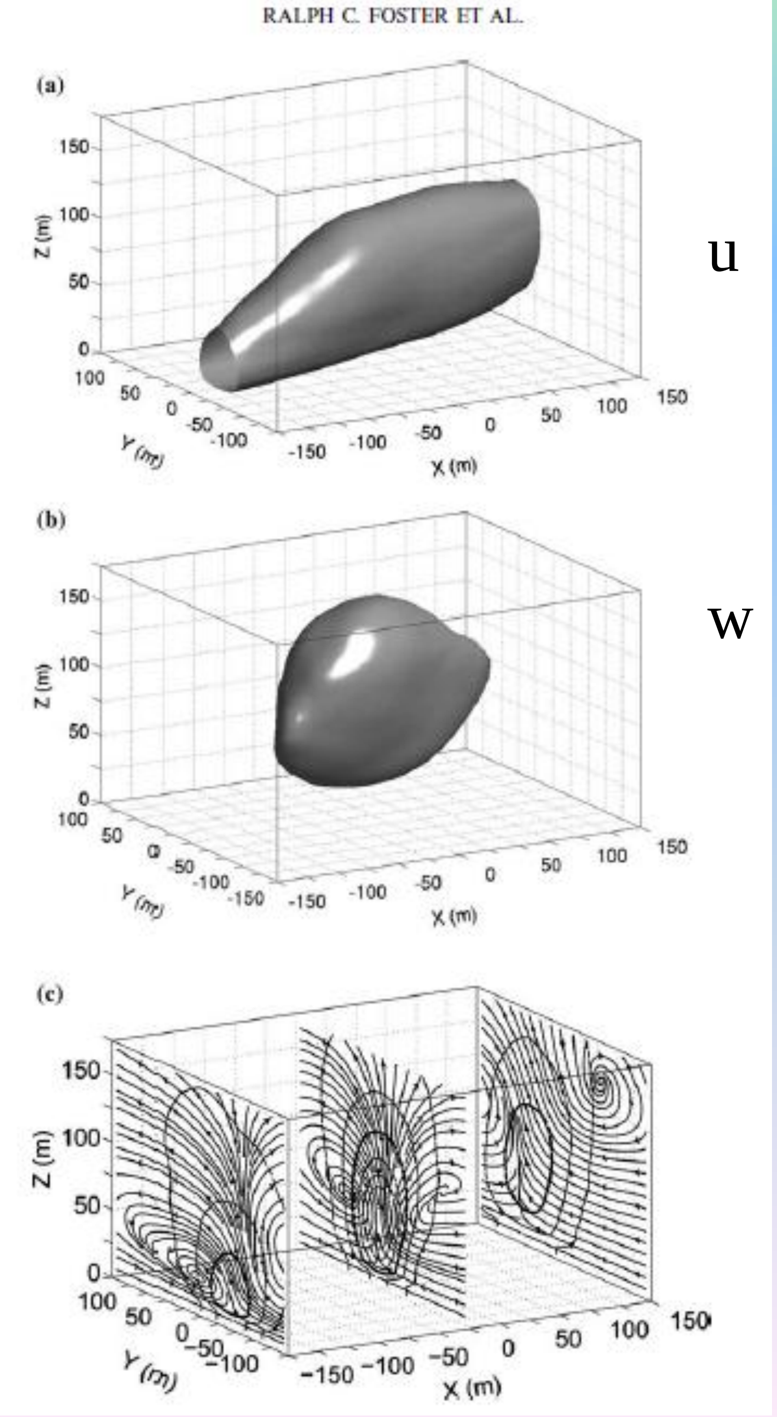


Optimization time

Optimization time

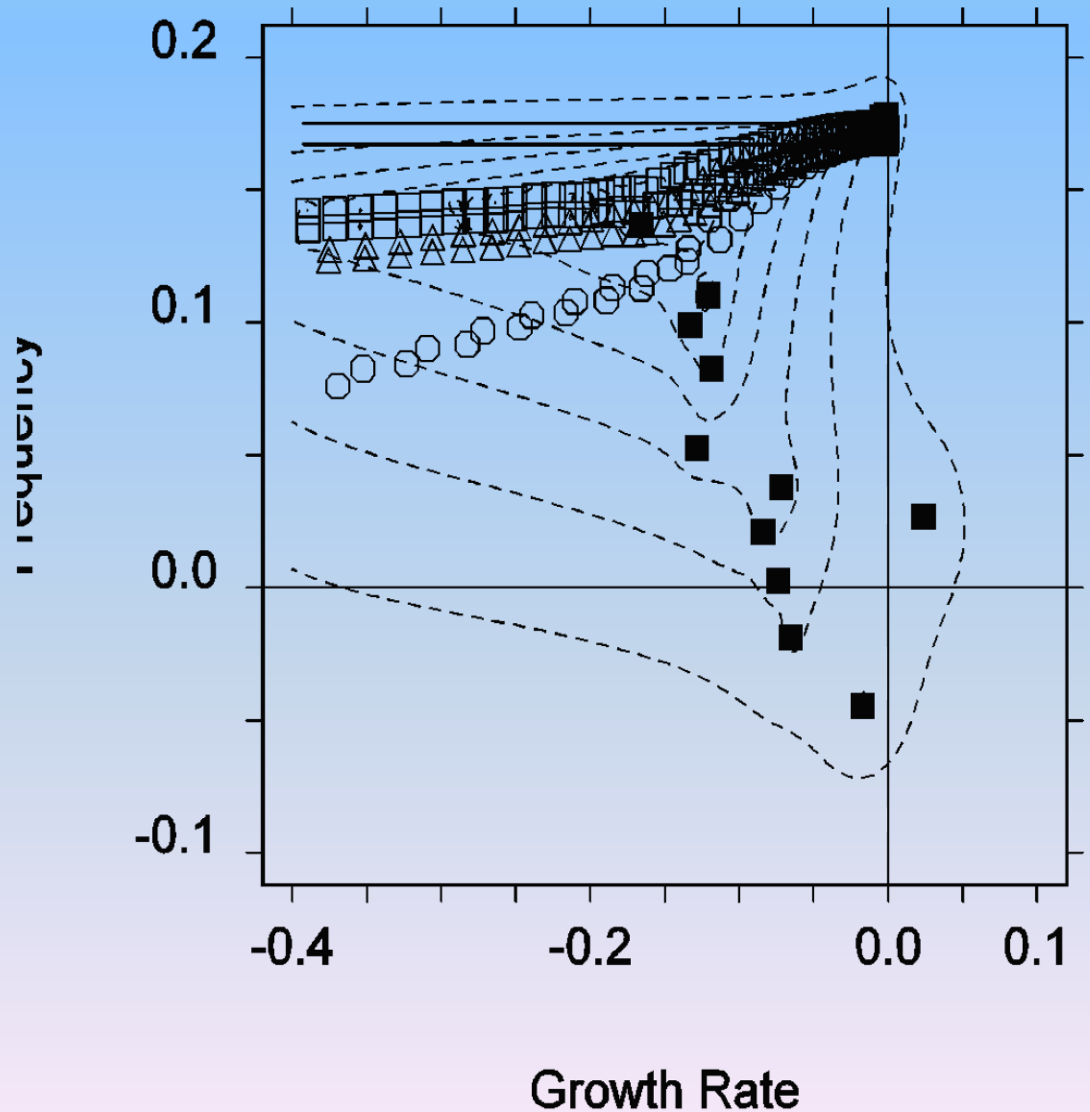
Projects on a roll mode, which dominates over time

No projection on roll mode



Conditionally-sampled ejection
Embedded in streak updraft

Boundary-Layer Meteorology (2006) **120**: 229–255


0.2
0.1
0.0
-0.1
-0.4
-0.2
0.0
0.1
Growth Rate

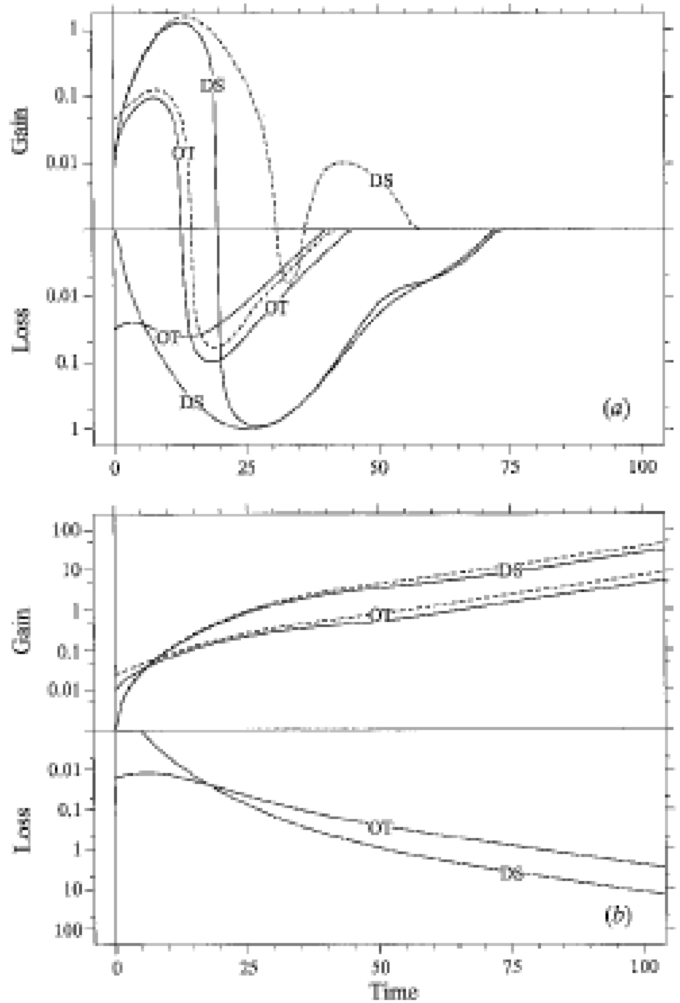


FIGURE 11. Vertically integrated kinetic energy budget terms as a function of time for the (*a*) $\tau = 15$ and (*b*) $\tau = 75$ optimal perturbations. The Coriolis terms cancel between the overturning (OT) and downstream (DS) budget terms and are omitted for clarity. The plotted terms are: energy growth, solid; shear production, dashed; dissipation, dash-dot.

Summary

- Rolls are associated with nonlinearly equilibrated normal modes
  - Over long times, normal modes maximize difference between growth and dissipation
  - Nonlinear effects stabilize
  - Modified mean flow & ***non-local*** fluxes
- Streaks are related to transient perturbations
  - Continuous cycle of formation→ growth → decay → reformation
  - Can have larger separation between growth and dissipation over short times
  - Flux events (ejections/sweeps) form in streak up-/down-drafts
  - Can co-exist with Rolls
  - Can energize roll modes