

# Improving Eyewall/Rainband In-Cloud Turbulent Mixing Parameterization in HWRF

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NOAA, HRD, EMC, NOAA

# Outlines

1. A brief discussion of TC intensification by eyewall/rainband eddy forcing
2. A short review on our previous attempt and recent progress on improving eyewall/rainband in-cloud turbulent mixing parameterization in HWRF.
3. Preliminary results.
4. Summary

## Azimuthal-mean tangential wind budget equation in a cylindrical coordinate

$$\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial r} + \bar{w} \frac{\partial \bar{v}}{\partial z} = \bar{u} \left( f + \frac{\bar{v}}{r} \right) + F_\lambda + F_{sgs_\lambda}, \text{ where}$$

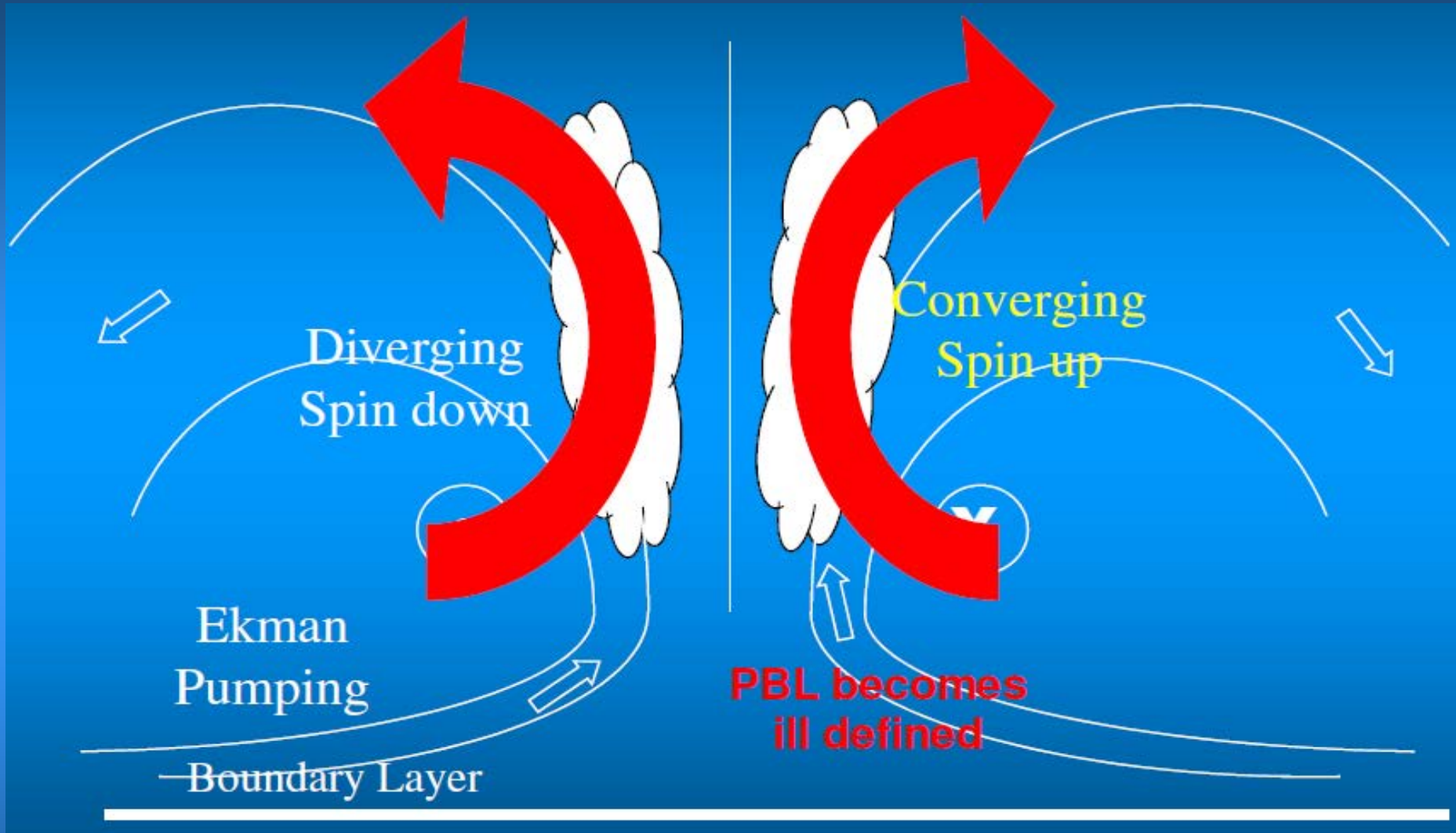
$$F_\lambda = -\overline{u' \frac{\partial v'}{\partial r}} - \overline{v' \frac{\partial v'}{r \partial \lambda}} - \overline{w' \frac{\partial v'}{\partial z}} - \frac{\overline{u' v'}}{r} : \text{model-resolved eddy forcing}$$

$F_{sgs_\lambda}$  : parameterized sub-grid scale (SGS) eddy forcing

$$\frac{D\bar{M}}{Dt} = \frac{\partial \bar{M}}{\partial t} + \bar{u} \frac{\partial \bar{M}}{\partial r} + \bar{w} \frac{\partial \bar{M}}{\partial z} = r(F_\lambda + F_{sgs_\lambda}),$$

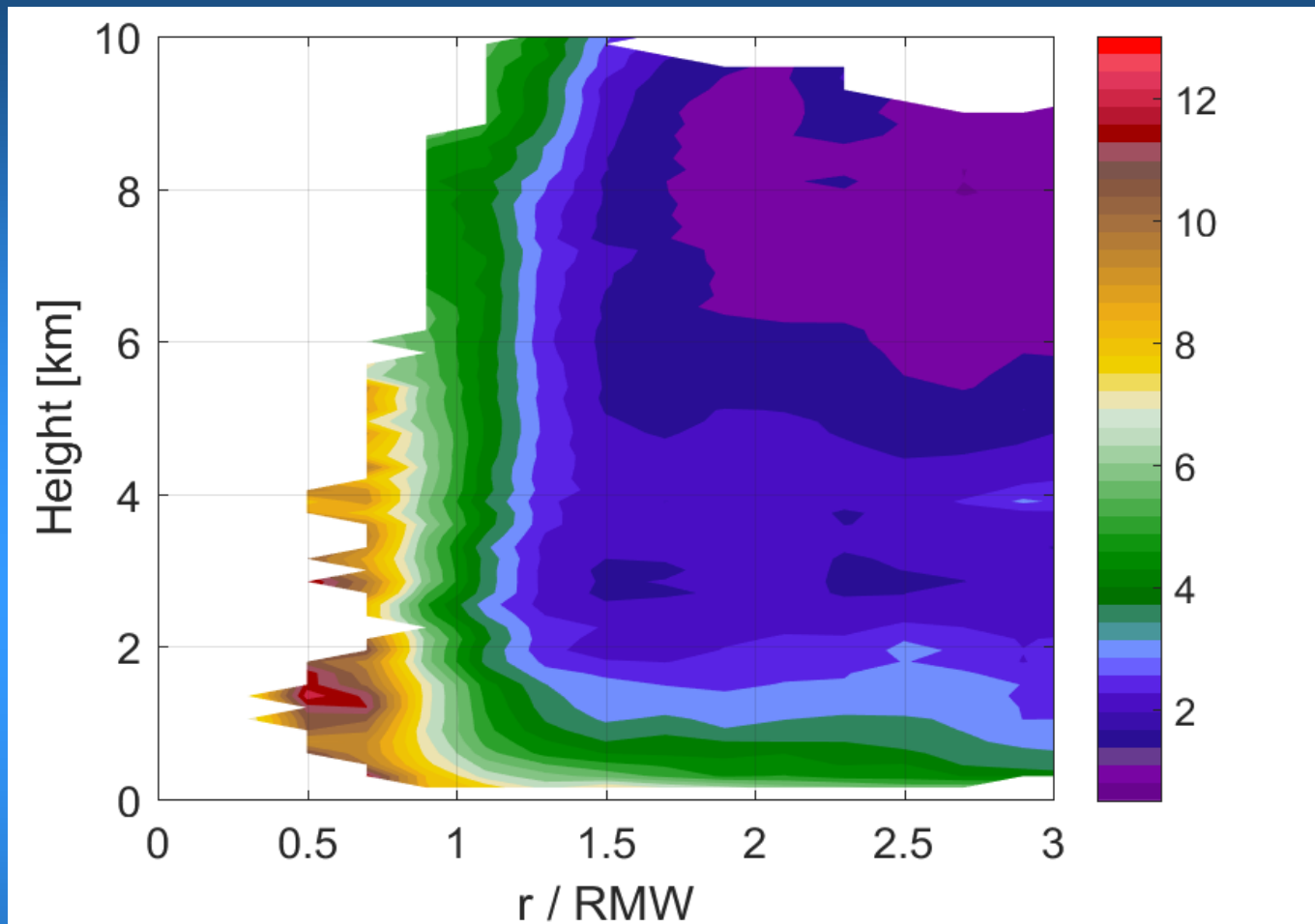
where  $\bar{M} = r\bar{v} + \frac{1}{2}fr^2$  is the azimuthal-mean absolute angular momentum

Higher model resolution allows the eddy forcing to be better resolved. The uncertainty arises from the parametrical determination of SGS eddy forcing.



In all TC theories, the boundary layer is treated as a shallow turbulent layer adjacent to Earth's surface with a depth typically about 1 km. But such a classic view of turbulent boundary layer creates a problem in the eyewall!

# Composite TKE derived from 116 radial legs of P3 flights in 2003-2010 seasons



The question here is not all about how to redefine boundary layer to encompass all the scenarios including the deep convective regime. The real question is: how to appropriately parameterize SGS eddy forcing both within and above the boundary layer?

In models, the SGS eddy forcing is determined by the turbulent mixing scheme.

The HWRF turbulent mixing scheme is a typical K-closure scheme

1. Within the PBL ( $z < h$ ),  $K_m = \kappa \frac{u_*}{\phi_m} \alpha z \left(1 - \frac{z}{h}\right)^2$ ,

$$h = Ri_{cr} \frac{\theta_{va} |U(h)|^2}{g[\theta_v(h) - \theta_s]}$$

2. Above the PBL ( $z \geq h$ ),

$$K_{m,t} = l^2 f_{m,t}(Ri_g) \sqrt{\left|\frac{\partial \bar{u}}{\partial z}\right|^2 + \left|\frac{\partial \bar{v}}{\partial z}\right|^2},$$

$l$  is the mixing length.

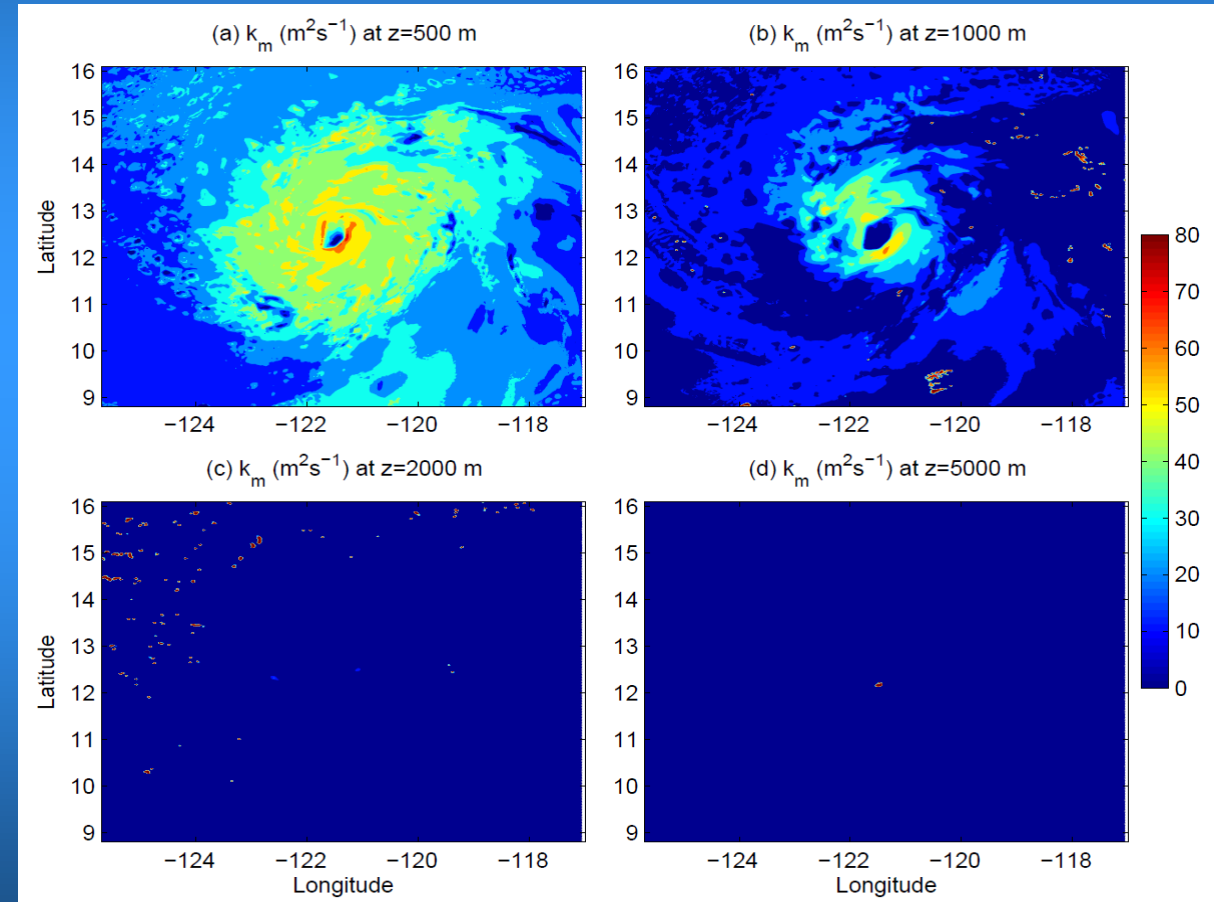
$f_{m,t}(Ri_g)$  is a stability function of gradient Richardson number.

$$Ri_g = N^2 / \left( \left|\frac{\partial \bar{u}}{\partial z}\right|^2 + \left|\frac{\partial \bar{v}}{\partial z}\right|^2 \right).$$

Brunt-Vaisala frequency,  $N^2 = \frac{g}{\theta_0} \frac{\partial \bar{\theta}_v}{\partial z}$

**A very poor estimate of static stability in clouds!**

Eddy exchange coefficients from a HWRF simulation of Hurricane Jimena (2015) at different altitudes at 12:00 UTC, 28 August, 2015

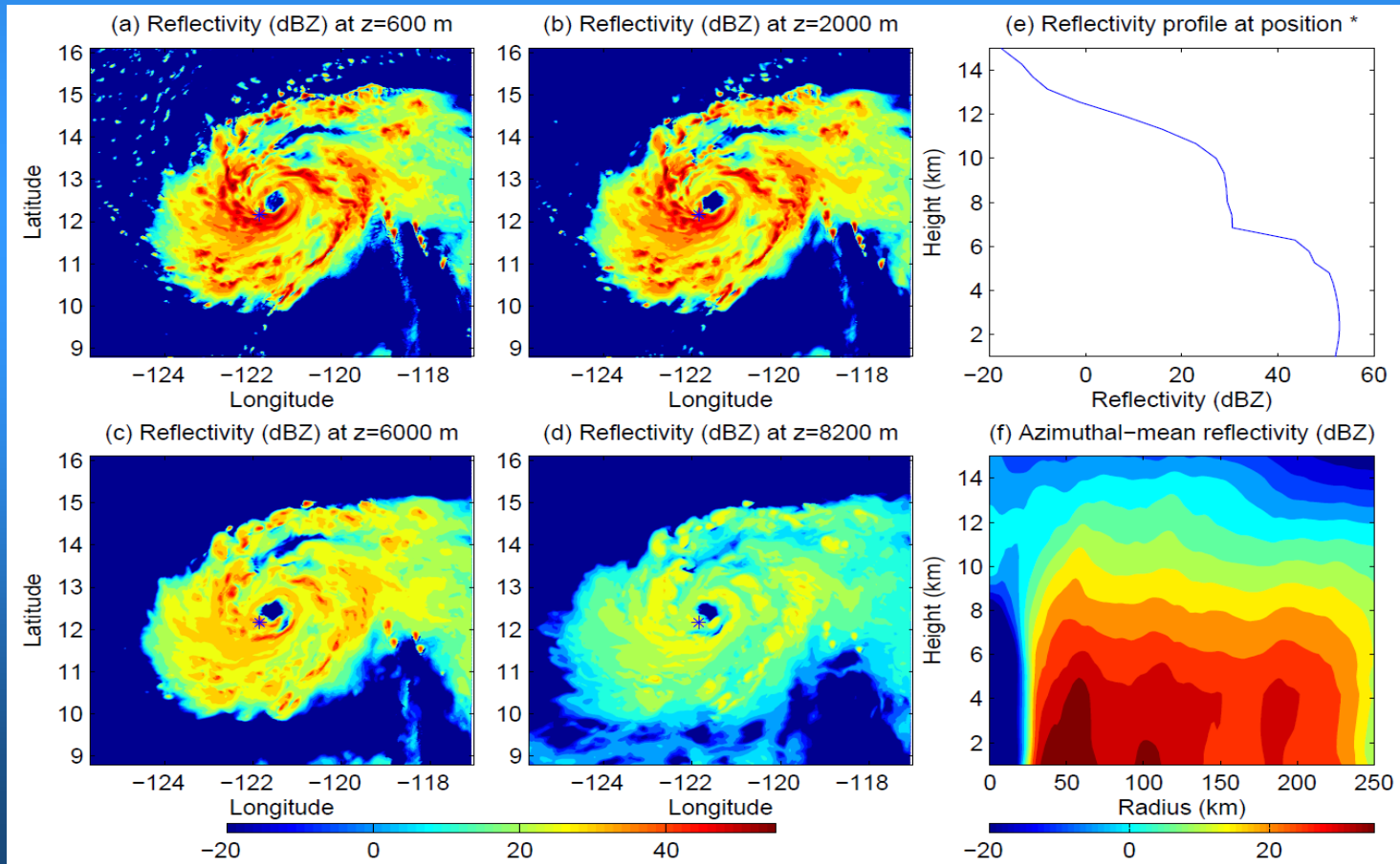


We hypothesize that lack of appropriate SGS eddy forcing associated with eyewall/rainband convection above the PBL is one of the culprits for the intensity forecast failure in many cases.

# Improving in-cloud turbulent mixing parameterization

Method 1: Treating the turbulence generated by surface processes and cloud processes as a whole, i.e., parameterizing the turbulence in the eyewall and rainbands based on the integrated “Turbulent Layer (TL)”

$$K_m = \kappa \frac{u_*}{\phi_m} \alpha z \left(1 - \frac{z}{h}\right)^2, \quad h = \text{TL}$$



Using radar reflectivity to determine TL

Criticism: a much larger  $h$  may affect turbulence parameterization near surface

Method 2: Keeping the diagnosed boundary layer height the same,

$$K_{m,t} = l^2 f_{m,t} (Ri_g) \sqrt{\left| \frac{\partial \bar{u}}{\partial z} \right|^2 + \left| \frac{\partial \bar{v}}{\partial z} \right|^2}, \quad z \geq h, \quad l \text{ is the mixing length.}$$

$$Ri_g = N^2 / \left( \left| \frac{\partial \bar{u}}{\partial z} \right|^2 + \left| \frac{\partial \bar{v}}{\partial z} \right|^2 \right),$$

but aiming to provide more accurate estimate of static stability in clouds, i.e., calculating the Brunt-Vaisala frequency,  $N^2$ , more accurately.

This method was adapted by the YSU PBL scheme to treat turbulent mixing above the boundary layer. But the formula that they used to include the cloud effects on Brunt-Vaisala frequency was inappropriate.

Durran & Klemp (1982) derived an accurate expression of Brunt-Vaisala frequency for saturated atmosphere using parcel theory.  $a = -g \frac{\rho_p - \rho_e}{\rho_e}$  (Acceleration of a parcel)

Taking the Taylor series of  $\rho_p$  and  $\rho_e$  for  $\delta z \rightarrow 0$ ,  $a = -g \left\{ \left( \frac{d \ln \rho}{dz} \right)_p - \left( \frac{d \ln \rho}{dz} \right)_e \right\} \delta z = N_m^2 \delta z$

$$P = \rho R_d T \frac{1+q_s/\epsilon}{1+q_t}, \quad \frac{dp}{dz} = -\rho g, \quad \frac{dq_s}{dz} = \left( 1 + \frac{q_s}{\epsilon} \right) \left( \frac{L q_s}{R_v T^2} \frac{dT}{dz} - \frac{q_s}{p} \frac{dp}{dz} \right).$$



$$N_m^2 \approx g \left\{ \frac{1+B}{1+A} \left[ \frac{d \ln \theta}{dz} + \frac{1}{c_p T} \left( 1 + \frac{q_s}{\varepsilon} \right) \left( A C_p \frac{dT}{dz} - B g \right) \right] - \frac{dq_t}{dz} \right\}, \quad A = \frac{L^2 q_s}{c_p R_v T^2}, \quad B = \frac{L q_s}{RT}.$$

YSU made a couple of inappropriate assumptions:

(1), they dropped  $\frac{dq_t}{dz}$ .  $\frac{dq_t}{dz} = \frac{dq_s}{dz} + \frac{dq_c}{dz}$ , **Not a small term!**

(2), they assumed  $\frac{dT}{dz} = -\frac{g}{c_p}$ , **Apparently incorrect in the clouds!**

$$\longrightarrow N_m^2 \approx (1+B) \left[ N^2 - \frac{g^2}{c_p T} \left( \frac{A-B}{1+A} \right) \right], \quad N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}. \quad (\text{YSU formula})$$

For  $T \propto [300K, 240K] \rightarrow A \propto [3.69, 0.06], B \propto [0.71, 0.009],$

$$\frac{g^2}{c_p T} \left( \frac{A-B}{1+A} \right) \propto [2.02, 0.19] \times 10^{-4}.$$

In the eyewall  $N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}, \propto [\pm 0, 1 \times 10^{-4}]$

**The YSU formula significantly over-reduces the static stability in clouds!**

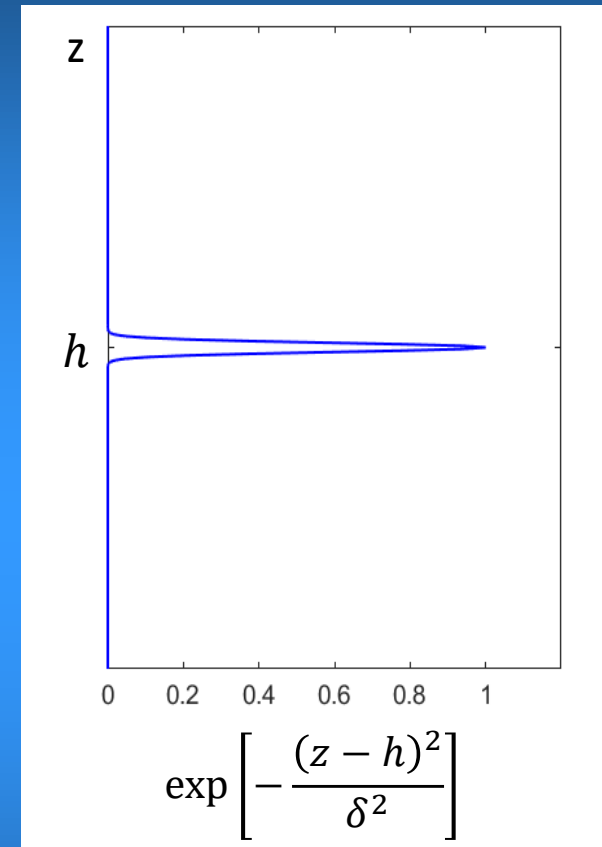
Because of significantly over-reduced Brunt-Vaisala frequency in clouds, it generates unrealistically large  $K_{m,t}$ . What YSU did is to artificially reduce  $K_{m,t}$  by averaging in-cloud  $K_{m,t}^{cld}$  and entrainment  $K_{m,t}^{ent}$ ,

$$K_{m,t} = (K_{m,t}^{cld} \cdot K_{m,t}^{ent})^{1/2}$$

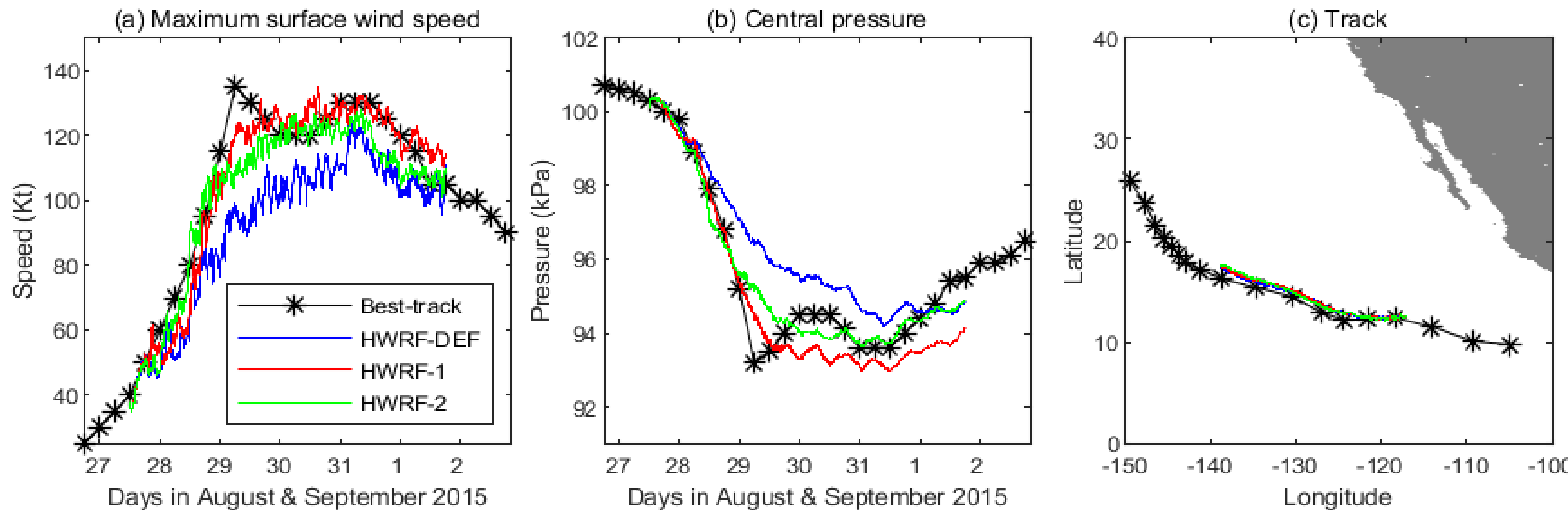
$$K_{m,t}^{ent} = Pr_{m,t} \frac{\overline{-w'\theta'_v}_h}{(\partial\theta_v/\partial z)_h} \exp\left[-\frac{(z-h)^2}{\delta^2}\right], \quad \frac{\delta}{h} \approx 0.02$$

In HWRF, we recalculated the Brunt-Vaisala frequency in clouds using accurate formula,

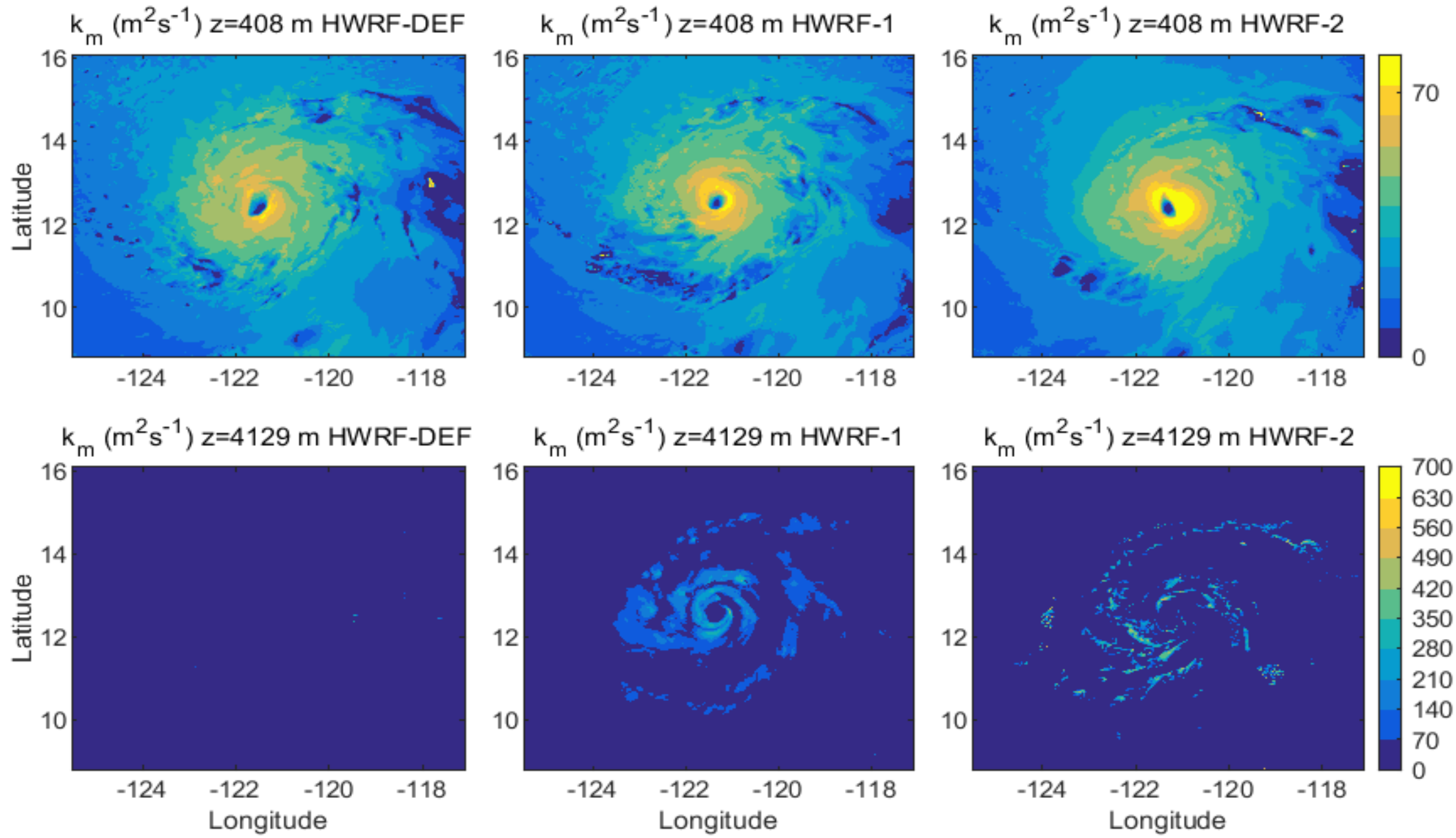
$$N_m^2 = g \left\{ \frac{1+B}{1+A} \left[ \frac{d\ln\theta}{dz} + \frac{L}{C_p T} \left( 1 + \frac{q_s}{\varepsilon} \right) \left( \frac{Lq_s}{R_v T^2} \frac{dT}{dz} + \frac{q_s g}{RT_v} \right) \right] - \frac{dq_t}{dz} \right\}$$



# Hurricane Jimena (2015)

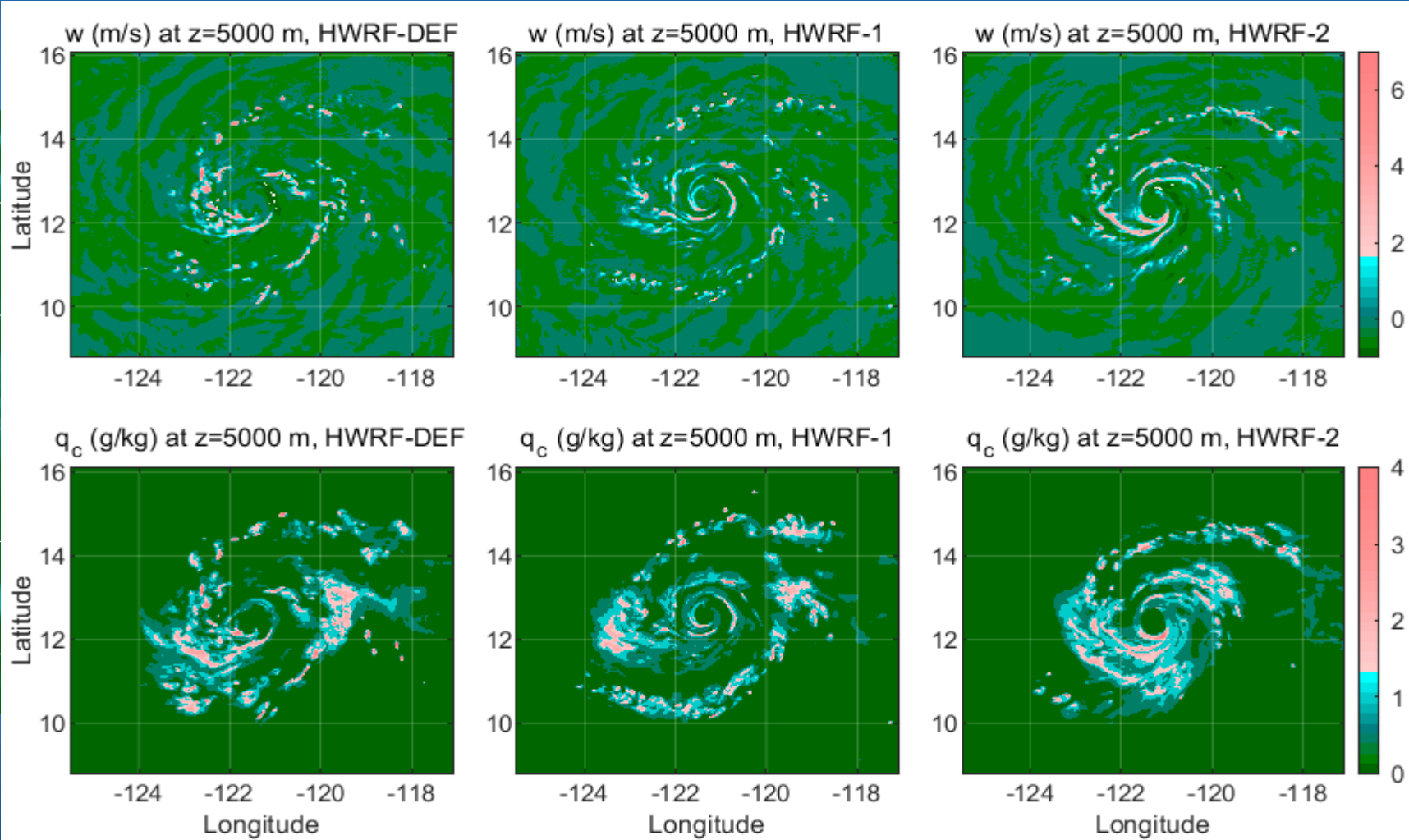
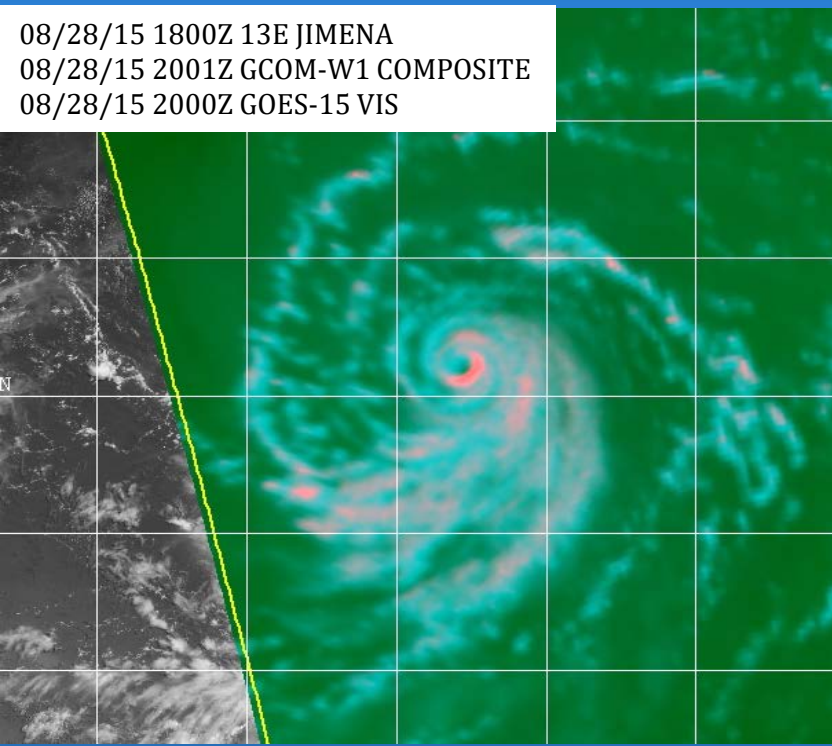


HWRF-1: parameterization of in-cloud turbulent mixing based on the TL concept  
HWRF-2: parameterization of in-cloud turbulent mixing by recalculating  $N^2$  in clouds



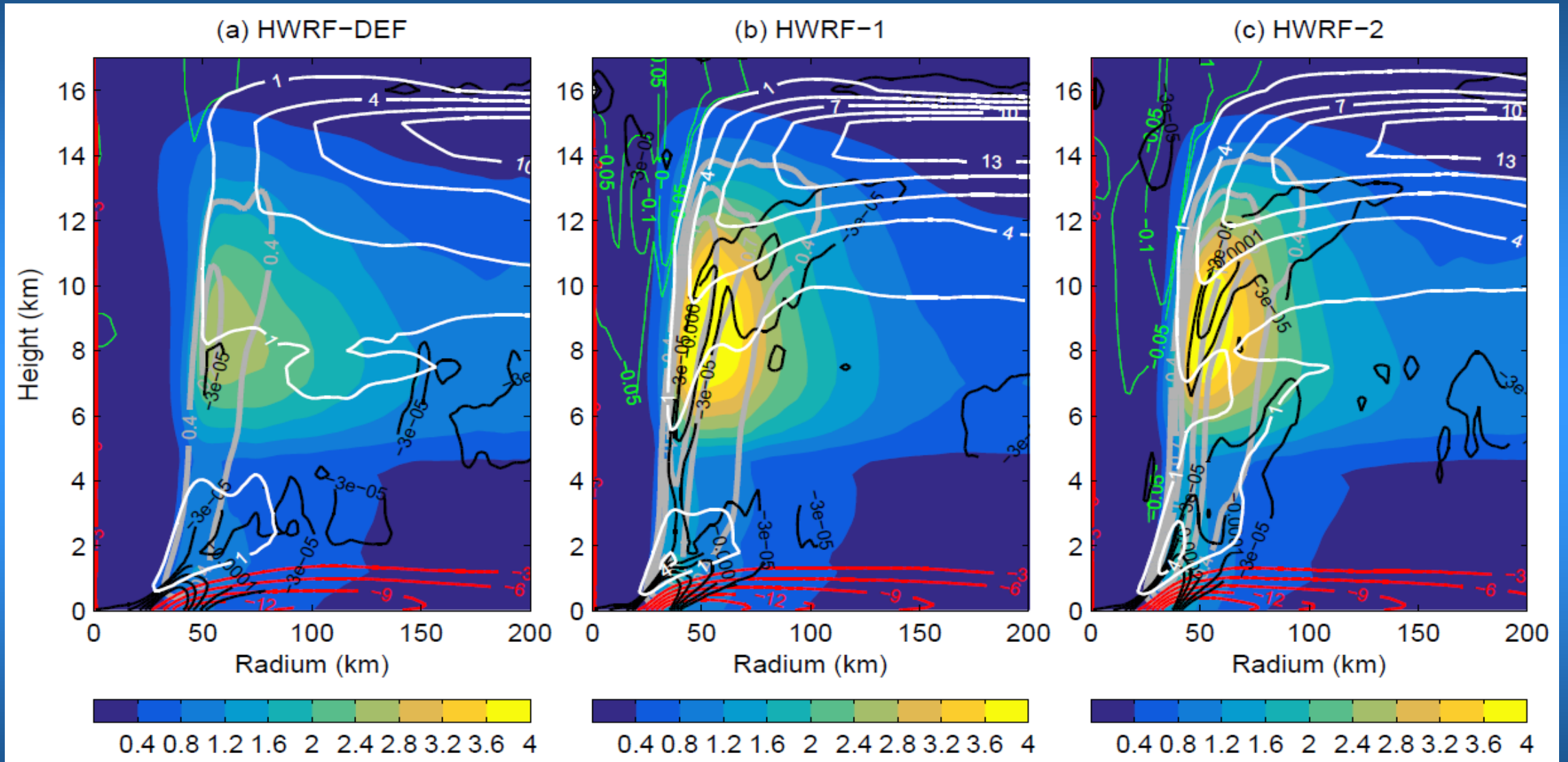
Eddy exchange coefficients at 12:00 UTC August 28, 2015

# Comparison of TC inner-core structure of Jimena (2015) right before Jimena's RI between satellite observations and three HWRf simulations.



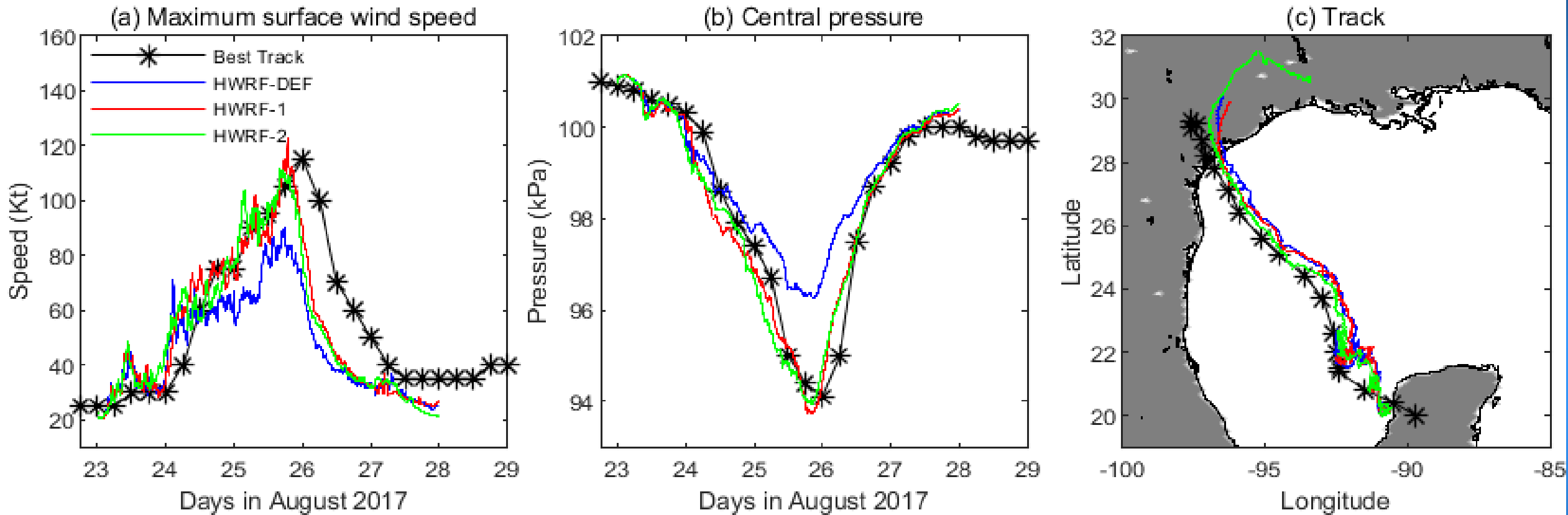
5-km vertical velocity and hydrometeor mixing ratio at 20:00 UTC August 28, 2015

Azimuthal-mean radius-height structure of Jimena (2015) simulated by three HWRFs averaged over the RI period from 12:00 UTC 08/28 to 06 UTC 08/29, 2015.

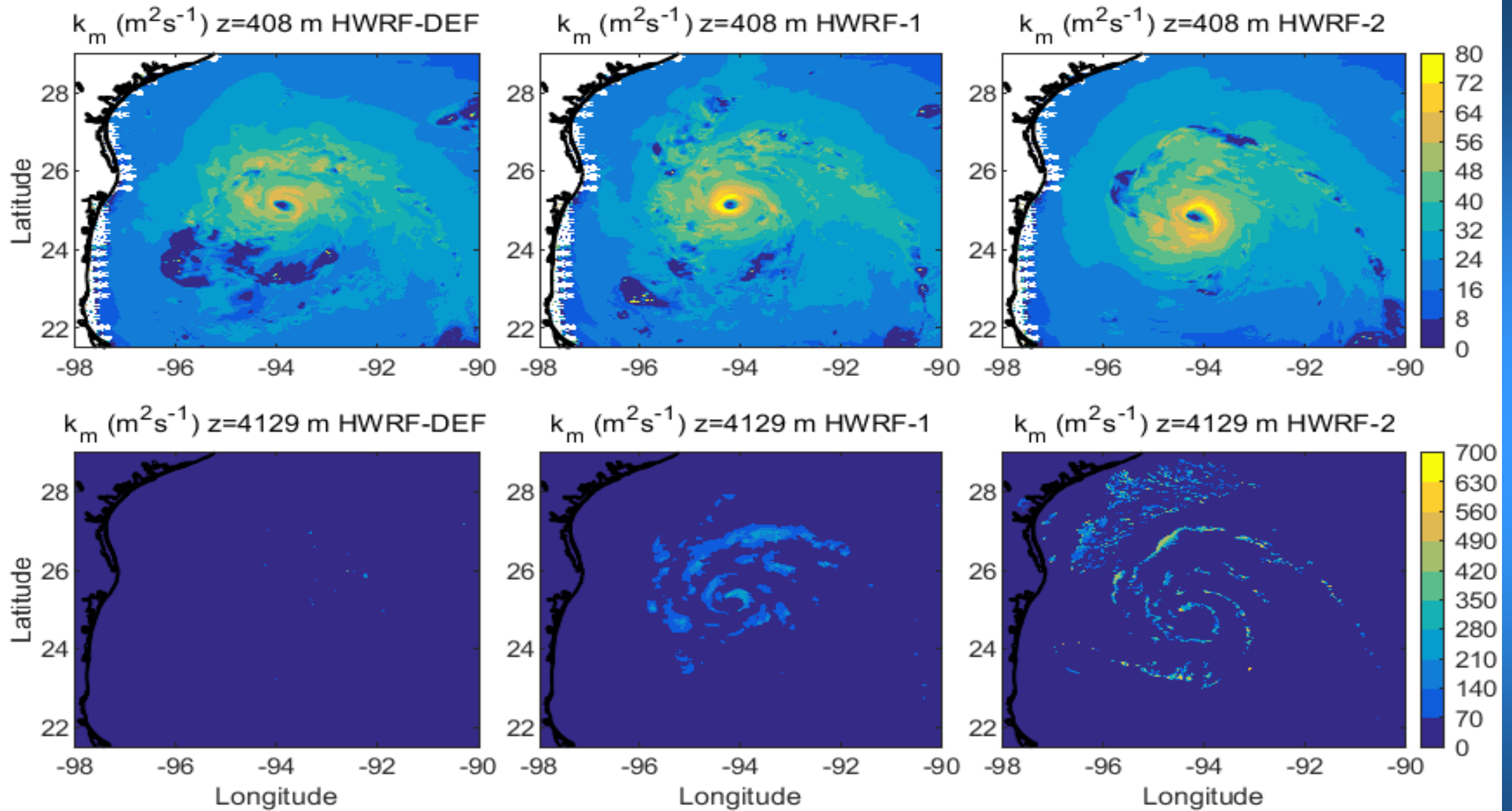


Hydrometeor mixing ratio (color shades), updrafts (gray contours), downdraft (green contours), radial inflow (red contours), and convergence of radial flow (black contours)

# Hurricane Harvey (2017)



HWRf-1: parameterization of in-cloud turbulent mixing based on the TL concept  
HWRf-2: parameterization of in-cloud turbulent mixing by recalculating  $N^2$  in clouds

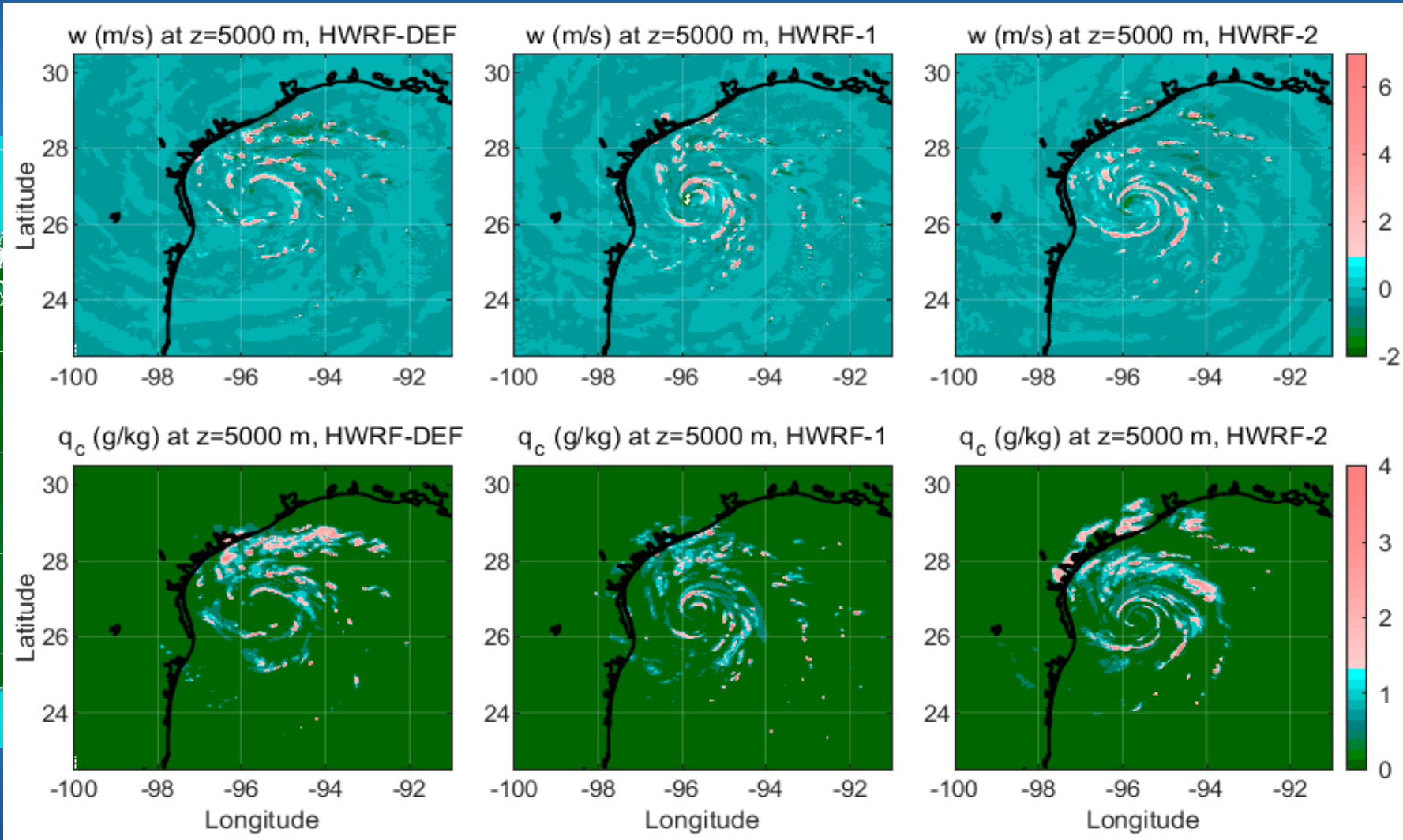
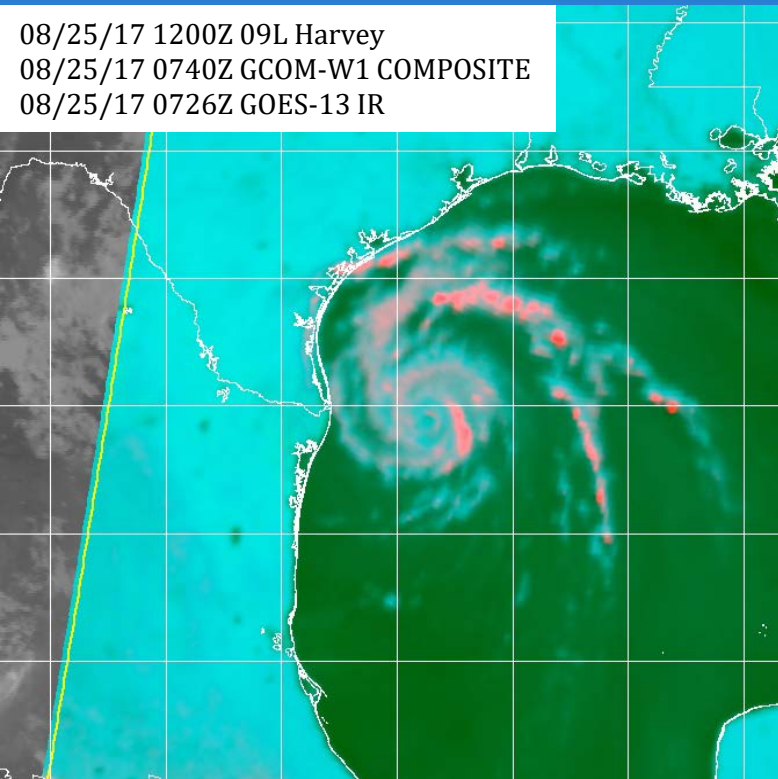


Eddy exchange coefficients at 18:00 UTC August 24, 2017



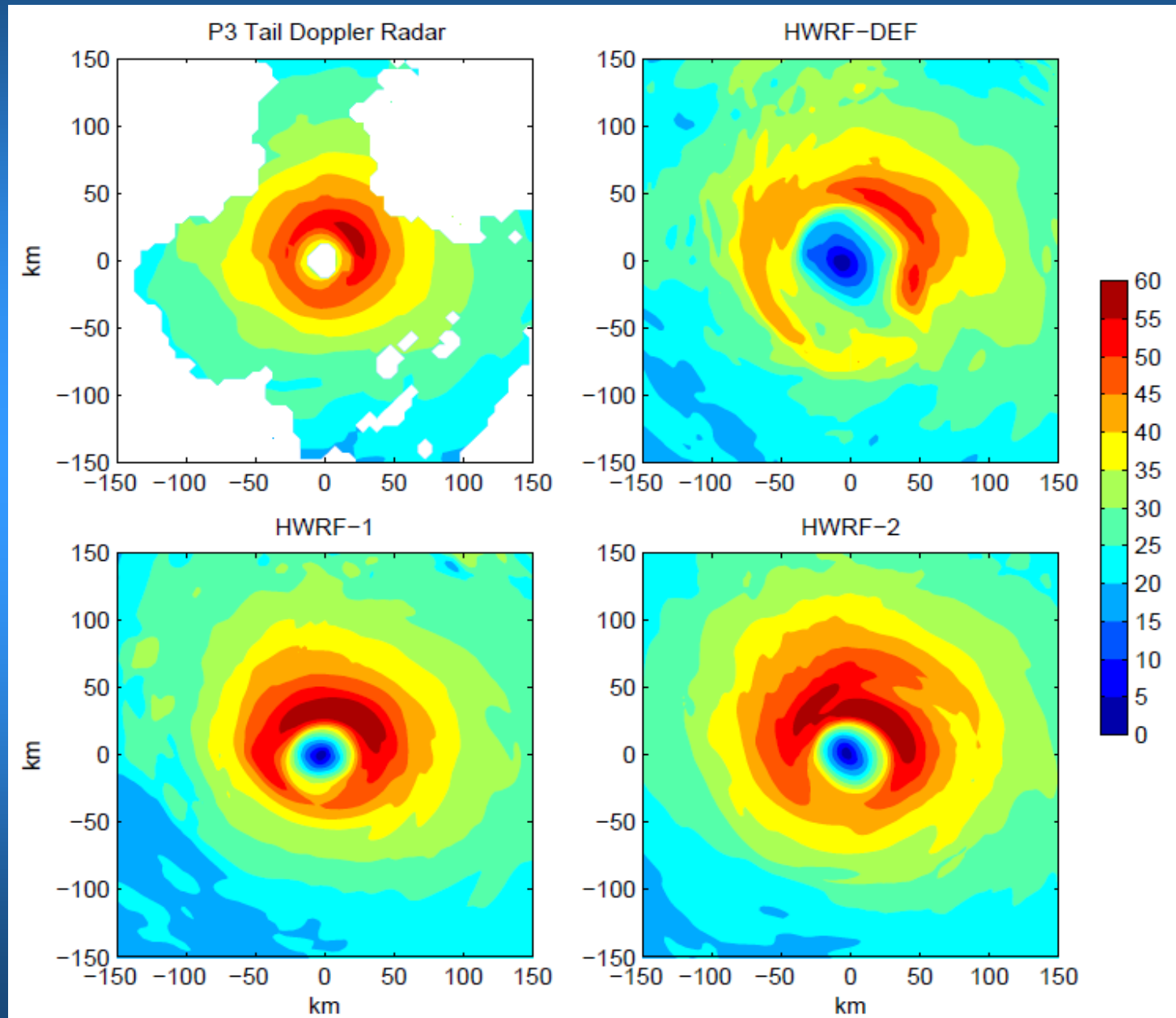
# Comparison of TC inner-core structure of Harvey (2017) between satellite observations and three HWRf simulations during Harvey's RI.

08/25/17 1200Z 09L Harvey  
08/25/17 0740Z GCOM-W1 COMPOSITE  
08/25/17 0726Z GOES-13 IR

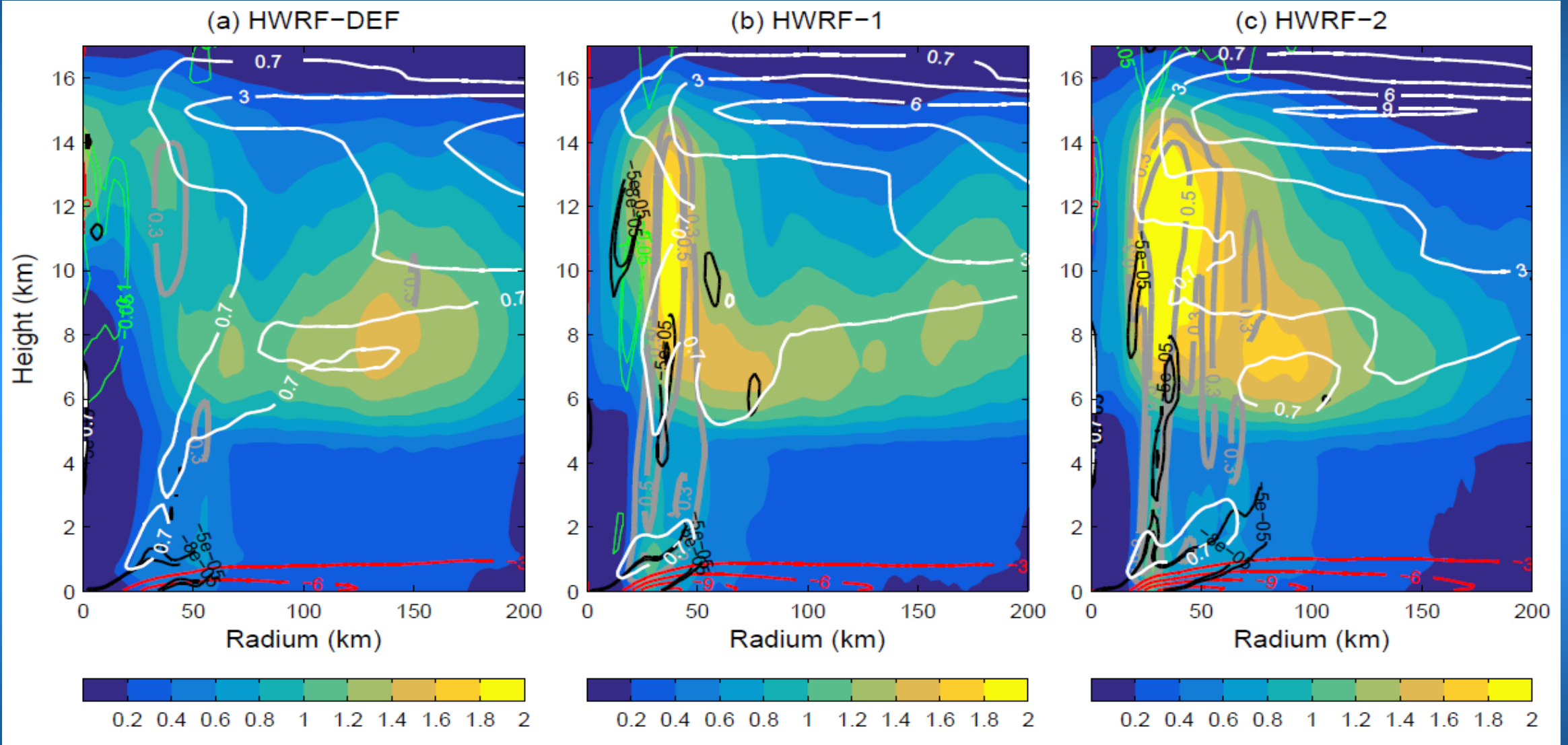


5-km vertical velocity and hydrometeor mixing ratio at 08:00 UTC August 25, 2017

# Comparison of 2.5-km wind speeds of Harvey (2017) at 18 UTC Aug. 25, 2017 between P3 tail Doppler radar observations and three HWRf simulations



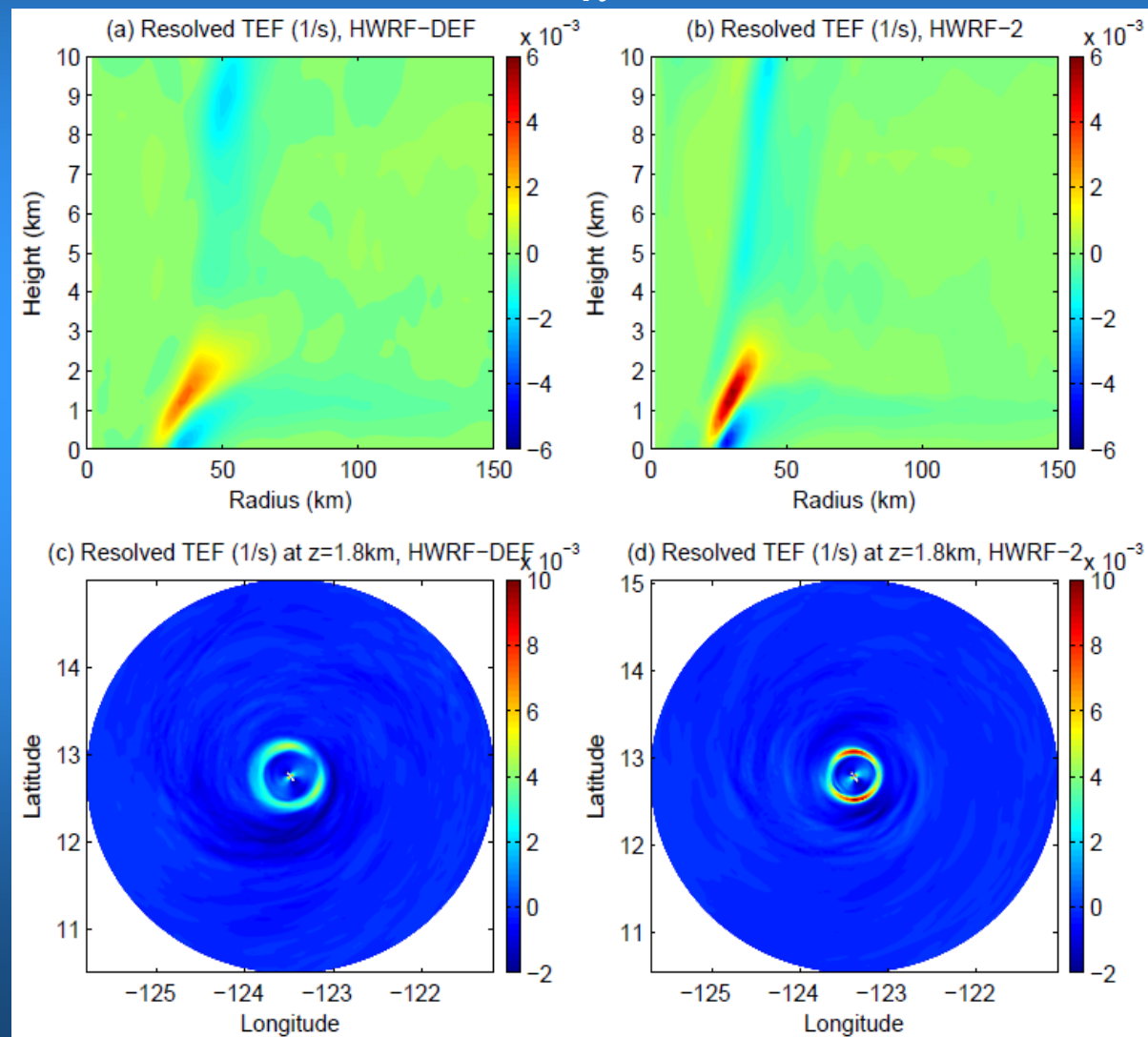
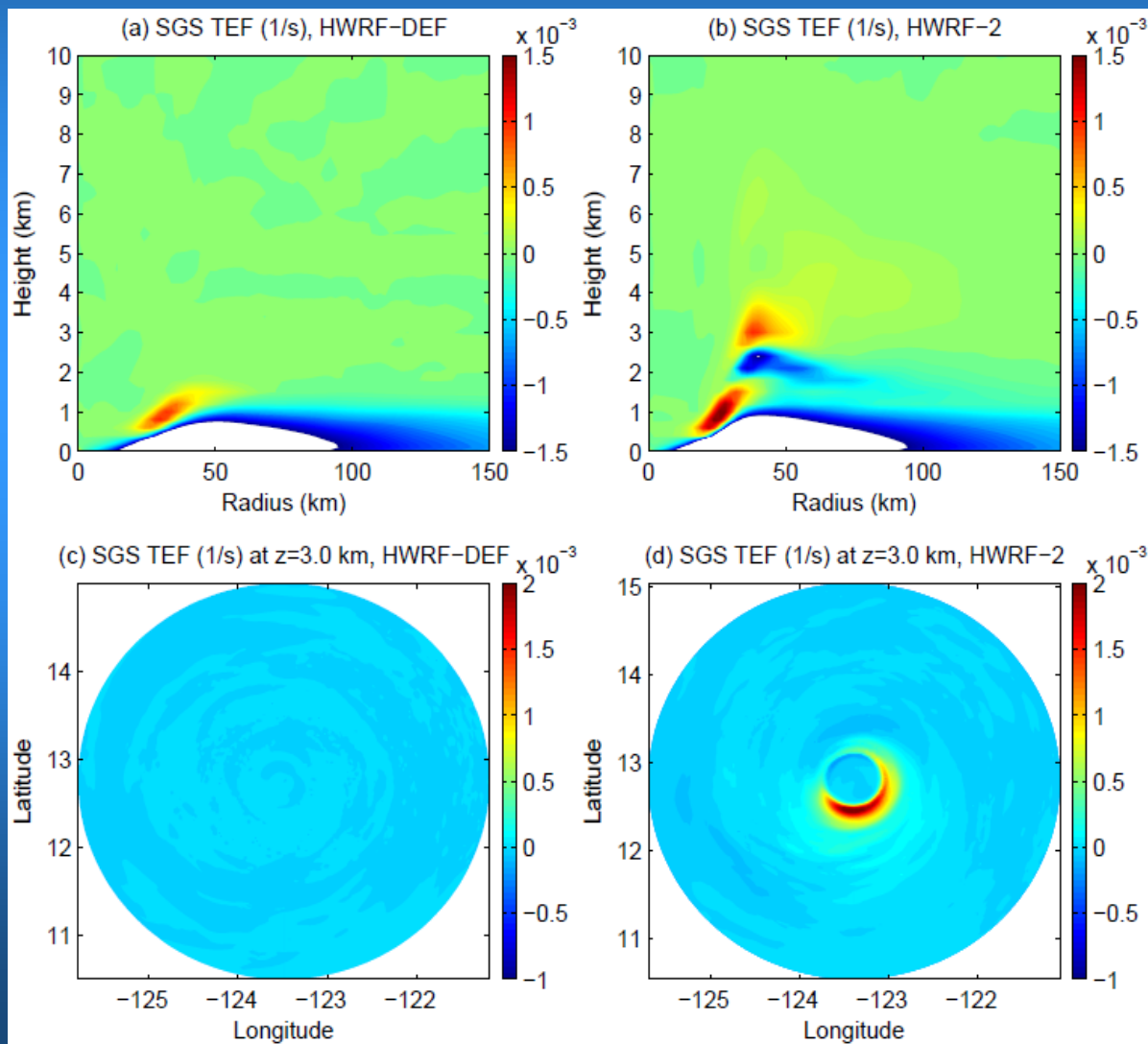
Azimuthal-mean radius-height structure of Harvey (2017) simulated by three HWRFs averaged over the RI period from 18:00 UTC 08/24 to 12 UTC 08/25, 2017.



Hydrometeor mixing ratio (color shades), updrafts (gray contours), downdraft (green contours), radial inflow (red contours), and convergence of radial flow (black contours)

# Budget analyses

$$\frac{\partial \bar{v}}{\partial t} = -\bar{u} \frac{\partial \bar{v}}{\partial r} - \bar{w} \frac{\partial \bar{v}}{\partial z} - \bar{u} \left( f + \frac{\bar{v}}{r} \right) + F_\lambda + F_{sgs\lambda}, \quad F_\lambda = -\overline{u' \frac{\partial v'}{\partial r}} - \overline{v' \frac{\partial v'}{r \partial \lambda}} - \overline{w' \frac{\partial v'}{\partial z}} - \frac{\overline{u' v'}}{r}.$$



# Summary

- A successful prediction of TC intensity depends on the skills of a model to generate eddy forcing that drives the primary and secondary circulations of a TC, provided that the model simulates correct large-scale fields and SST.
- While it is negative definite in the PBL, the sign of eddy forcing associated with eyewall/rainband convection above the PBL is indefinite. It can be positive depending on the detailed eddy processes, and thus, provides a mechanism to spin up a TC vortex.
- In numerical models, the eyewall/rainband eddy forcing with continuous spectra is artificially split into two parts: the model-resolved and SGS components. But they are not independent. While higher model resolution allows the eddy forcing to be better resolved, the SGS eddy forcing is a source of uncertainty. At the resolution of operational HWRF, the resolved eddy forcing and the associated storm inner-core structure show a substantial dependence on the SGS eddy forcing.
- With the correct determination of Brunt-Vaisala frequency in clouds, the HWRF PBL scheme is shown to have the ability to appropriately generate in-cloud turbulent eddy forcing in the eyewall and rainbands.