UCLA JIFRESSE

Joint Institute for Regional Earth System Science & Engineering

NASA

A robust representation of rain microwave radiances for data assimilation and to estimate the 1st few radial modes of heating, vertical motion, precipitable water, total ice and rain

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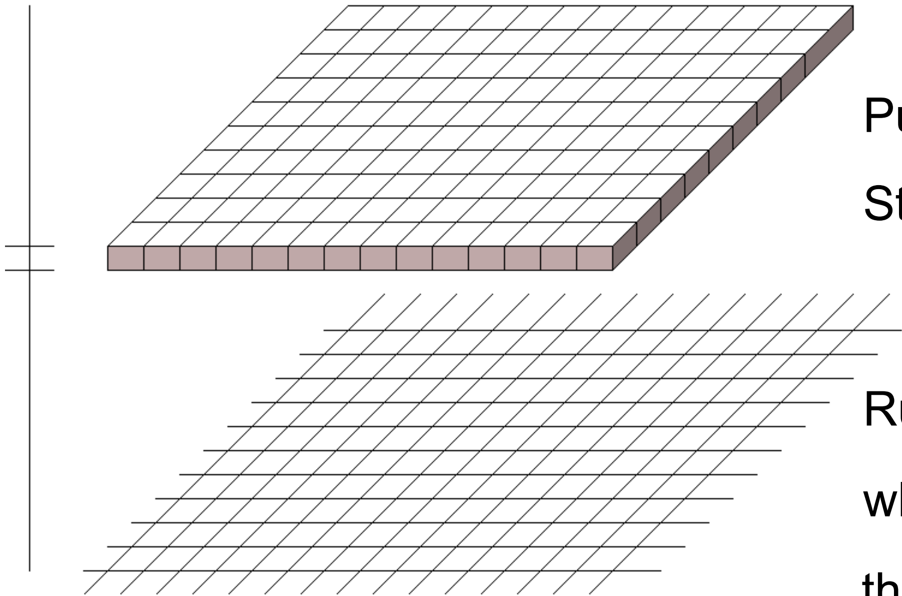


Unknowns (variables): in each volume element,
$x = (T, p, u, v, w, q_{wv}, q_{cl}, q_{pl}, q_{ci}, q_s, q_g, q_h)$

Put them all together in one **X** living in $\mathbf{R}^{126,000,000}$

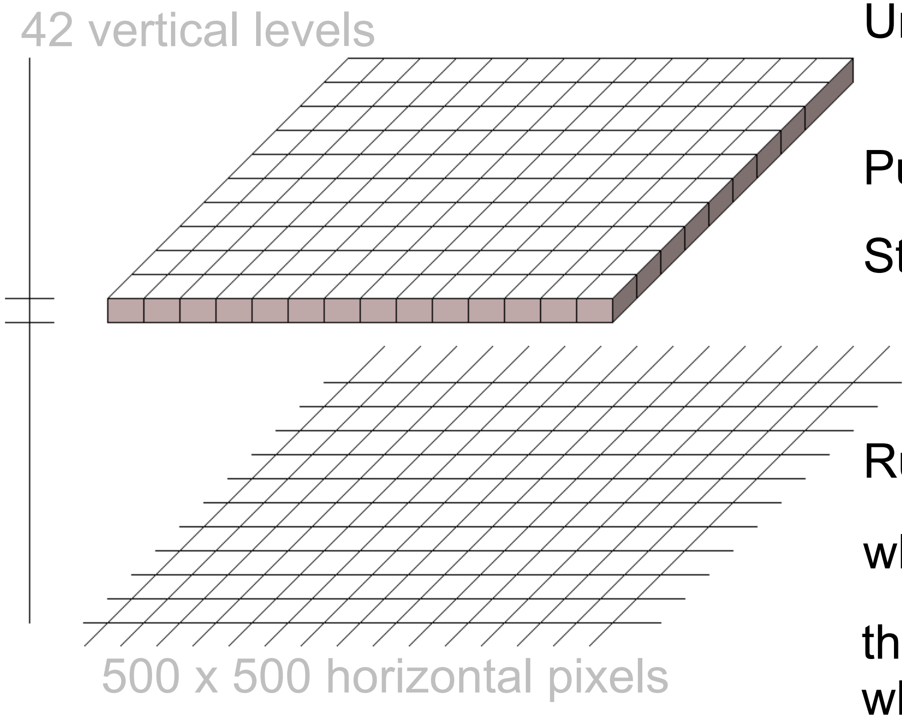
Start with $\mathbf{X} = \mathbf{X}_0$, initial condition known up to (Gaussian) error with imperfectly known higher moments (covariance)

Run dynamics $d\mathbf{X}_t = F(\mathbf{X}_t; \lambda)\, dt$

where F is nonlinear and has parameters,

then, at time s, observe $O = H(\mathbf{X}_s; \lambda')$ + error, where nonlinear H depends on parameters λ' whose dynamics are not known.

Goal: find "**X**" consistent with dynamics-only $\mathbf{X}_s$ and such that H(**X**) is consistent with O



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where F is nonlinear and has parameters,

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where nonlinear H depends on parameters λ'
whose dynamics are not known.

Goal: find "**X**" consistent with dynamics-only $\mathbf{X}_s$ and such that $H(\mathbf{X})$ is consistent with O

Parameters λ in the definition of the dynamics F: microphysics
partitioning between cloud liquid, precipitating liquid, "small ice" (50 μm) and large ice mean sizes

Parameters λ' in the definition of the observation H:
all of λ along with habits (snow, graupel, hail)

Observe $O = H(\mathbf{X}_s; \lambda')$ + error:

What does H look like for microwave obs in rain?

e.g. for radar (fewer integrals than – but similar to – passive):

$$\mathrm{H}(q) = \int_0^\infty \sigma_b(D;\lambda_H) n(q) D^\mu e^{-\Lambda D}\, e^{-\int_{z_q}^\infty \left[\int_0^\infty \sigma_e(D';\lambda_H(z)) n(q(z)) D'^{\mu(z)} e^{-\Lambda(z) D'} dD'\right] dz}\, dD$$

where

$$n(q) = \frac{6\,\Lambda^{\mu+4}}{\pi\rho\,\Gamma(\mu+4)}\, q$$

Observe O = H($\mathbf{X}_s$; λ') + error:

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scattering x-sec'n
of hy'meteors of diam *D*

where

$$n(q) = \frac{6\,\Lambda^{\mu+4}}{\pi \rho\, \Gamma(\mu+4)}\, q$$

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number concentration
of hy'meteors of diam D

where

$$n(q) \;=\; \frac{6\,\Lambda^{\mu+4}}{\pi\rho\,\Gamma(\mu+4)}\, q$$

Observe O = H($\mathbf{X}_s$; λ') + error:

What does H look like for microwave obs in rain?

e.g. for radar (fewer integrals than – but similar to – passive):

$$\mathrm{H}(q) = \int_0^{\infty} \sigma_b(D;\lambda_H) n(q) D^{\mu} e^{-\Lambda D} \, e^{-\int_{z_q}^{\infty}\left[\int_0^{\infty} \sigma_e(D';\lambda_H(z)) n(q(z)) D'^{\mu(z)} e^{-\Lambda(z) D'} dD'\right] dz} \, dD$$

extinction from hydrometeors
at height z

where

$$n(q) = \frac{6\,\Lambda^{\mu+4}}{\pi\rho\,\Gamma(\mu+4)}\, q$$

Observe $O = H(\mathbf{X}_s; \lambda')$ + error:

What does H look like for microwave obs in rain?

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$$\mathrm{H}(q) \;=\; \int_0^\infty \sigma_b(D;\lambda_H) n(q) D^\mu e^{-\Lambda D} \; e^{-\int_{z_q}^\infty \left[\int_0^\infty \sigma_e(D';\lambda_H(z)) n(q(z)) D'^{\mu(z)} e^{-\Lambda(z) D'} dD'\right] dz} \, dD$$

where

$$n(q) \;=\; \frac{6\,\Lambda^{\mu+4}}{\pi\rho\,\Gamma(\mu+4)}\, q$$

and Λ and μ are parameters that are not independent with q (see slides 39-48 at the end of the presentation)

and the habits depend discontinuously on q

Observe $O = H(\mathbf{X}_s; \lambda')$ + error:

What does H look like for microwave obs in rain?

e.g. for radar (fewer integrals than – but similar to – passive):

$$H(q) = \int_0^\infty \sigma_b(D;\lambda_H)\, n(q) D^{\mu} e^{-\Lambda D}\; e^{-\int_{z_q}^{\infty}\left[\int_0^\infty \sigma_e(D';\lambda_H(z))\, n(q(z)) D'^{\mu(z)} e^{-\Lambda(z) D'}\, dD'\right] dz}\, dD$$

~ passive emission

~ passive scattering+absorption

where

$$n(q) = \frac{6\,\Lambda^{\mu+4}}{\pi\rho\,\Gamma(\mu+4)}\, q$$

and Λ and μ are parameters that are not independent with q

(see slides past the end of today's presentation)

and the habits depend discontinuously on q

Observe $O = H(\mathbf{X}_s; \lambda')$ + error:

What does H look like for microwave obs in rain?

In the assimilation, as q is adjusted, H will have discrete jumps

Ideally, the representation should “smooth over” these artificial jumps

⇒ define H empirically by fitting its values calculated with many different (yet feasible) combos of λ'

Methodology

• Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z), using stream ψ, potential χ, P, T, RH, W, q_{cliq}, q_r, q_{cli}, q_s, q_g, q_h at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$

• for each of these 12million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$

• find the principal components $x_1', \ldots, x_{504}'$ (each is a linear combo of $x_1, \ldots, x_{504}$) and the principal components $T_1', \ldots, T_9'$ (each a linear combo of $T_1', \ldots, T_9'$)

• Then we will have to find combos of $x_1',\ldots, x_{504}'$ that correlate most with combos of $T_1',\ldots, T_9'$

• Say these combos are x_1'', x_2'', x_3'' and T_1'', T_2'', T_3'': we finally need to express the latter in terms of the former, in a differentiable way (to be able to compute derivatives)

Methodology

- Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z), using stream ψ, potential χ, P, T, RH, W, q_{cliq}, q_r, q_{cli}, q_s, q_g, q_h at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$
- for each of these 12million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$

- Step 1: find the principal components $x_1', \ldots, x_{504}'$
- Step 2: find the principal components $T_1', \ldots, T_9'$

- Step 3: find
  combos of $x_1', \ldots, x_{504}'$ that correlate most with combos of $T_1', \ldots, T_9'$

Methodology

- Start with HWRF simulations (say HEDAS Earl 2010 h3vk, 2010-08-29-12Z to 2010-09-03-18Z), using stream ψ, potential χ, P, T, RH, W, q_{cliq}, q_r, q_{cli}, q_s, q_g, q_h at 42 vertical levels for a total of 504 variables $x_1, \ldots, x_{504}$
- for each of these 12million columns, forward-calculate $T_{b1}, \ldots, T_{b9}$

- Step 1: find the principal components $x_1', \ldots, x_{504}'$
- Step 2: find the principal components $T_1', \ldots, T_9'$

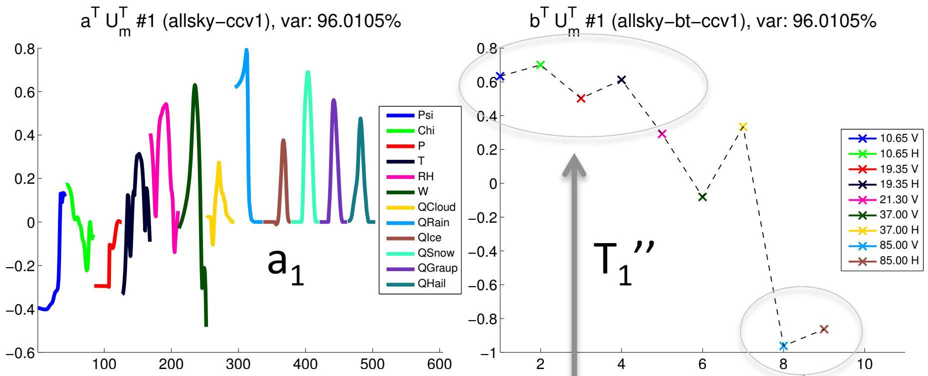
- Step 3: find combos of $x_1',\ldots, x_{504}'$ that correlate most with combos of $T_1',\ldots, T_9'$

  and express T_1'', T_2'', T_3'' in terms of x_1'', x_2'', x_3'' (with differentiable expression, in order to compute derivatives):

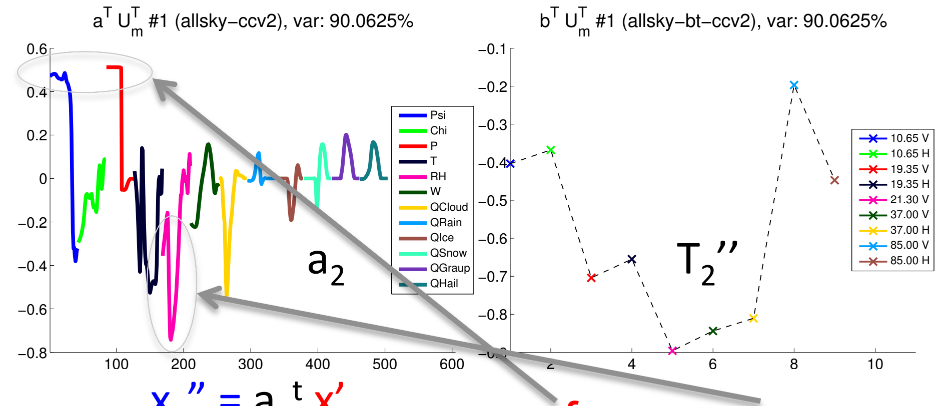
$$T_i''(x_1'', x_2'', x_3'') = \sum T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

where the weighted sum over n runs over the 12million training points

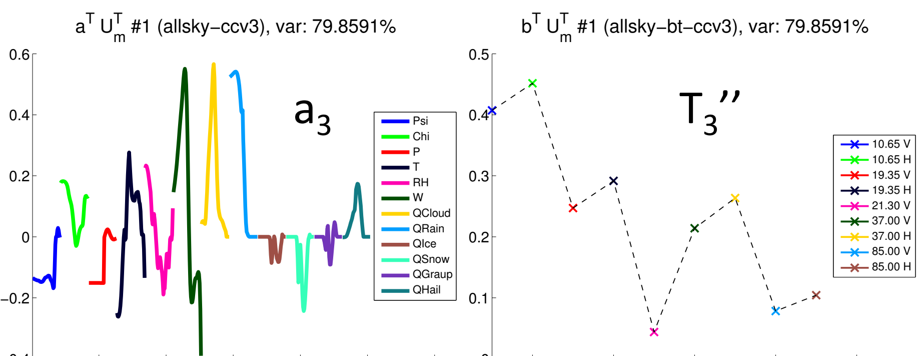
First part of step 3: here are the first 3 x'' and T''



$x_1'' = a_1^t x'$ emission – scattering

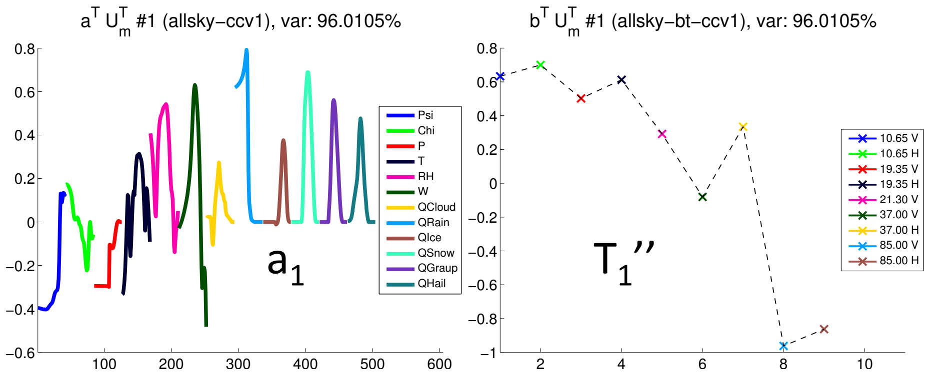


$x_2'' = a_2^t x'$ surface – vapor

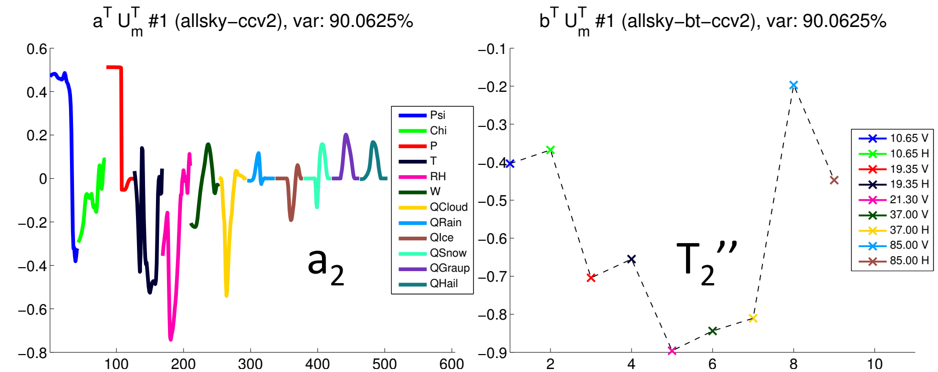


$x_3'' = a_3^t x'$ $T_3'' = b_3^t T'$

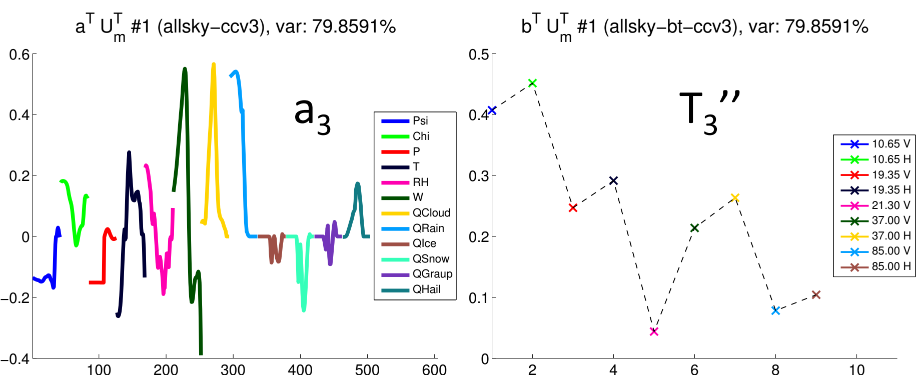
First part of step 3: here are the first 3 x'' and T''



$x_1'' = a_1^t\, x'$ emission – scattering



$x_2'' = a_2^t\, x'$ surface – vapor



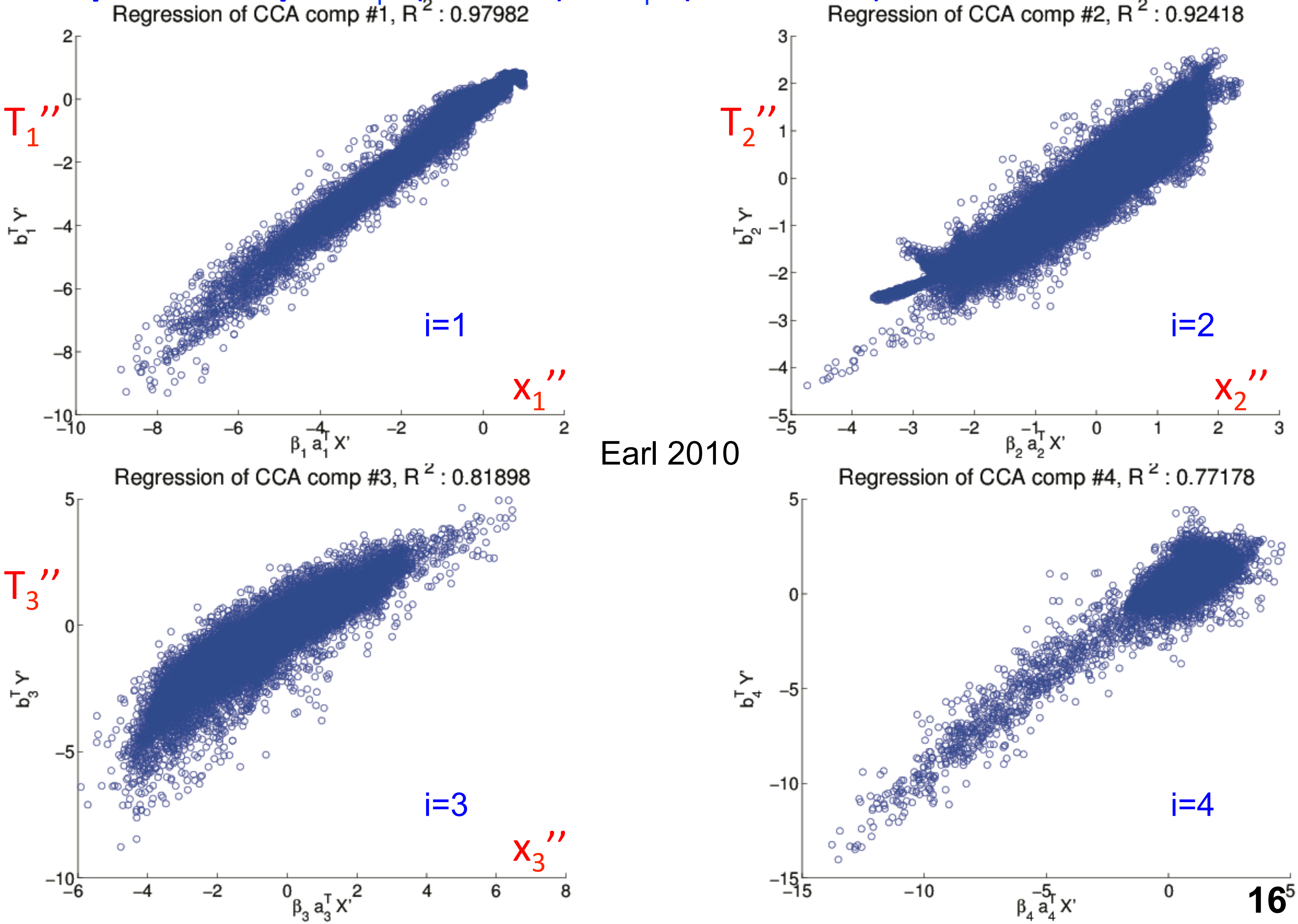
$x_3'' = a_3^t\, x'$ $T_3'' = b_3^t\, T'$

Most remarkable:
the operators H_1, H_2 , H_3
giving

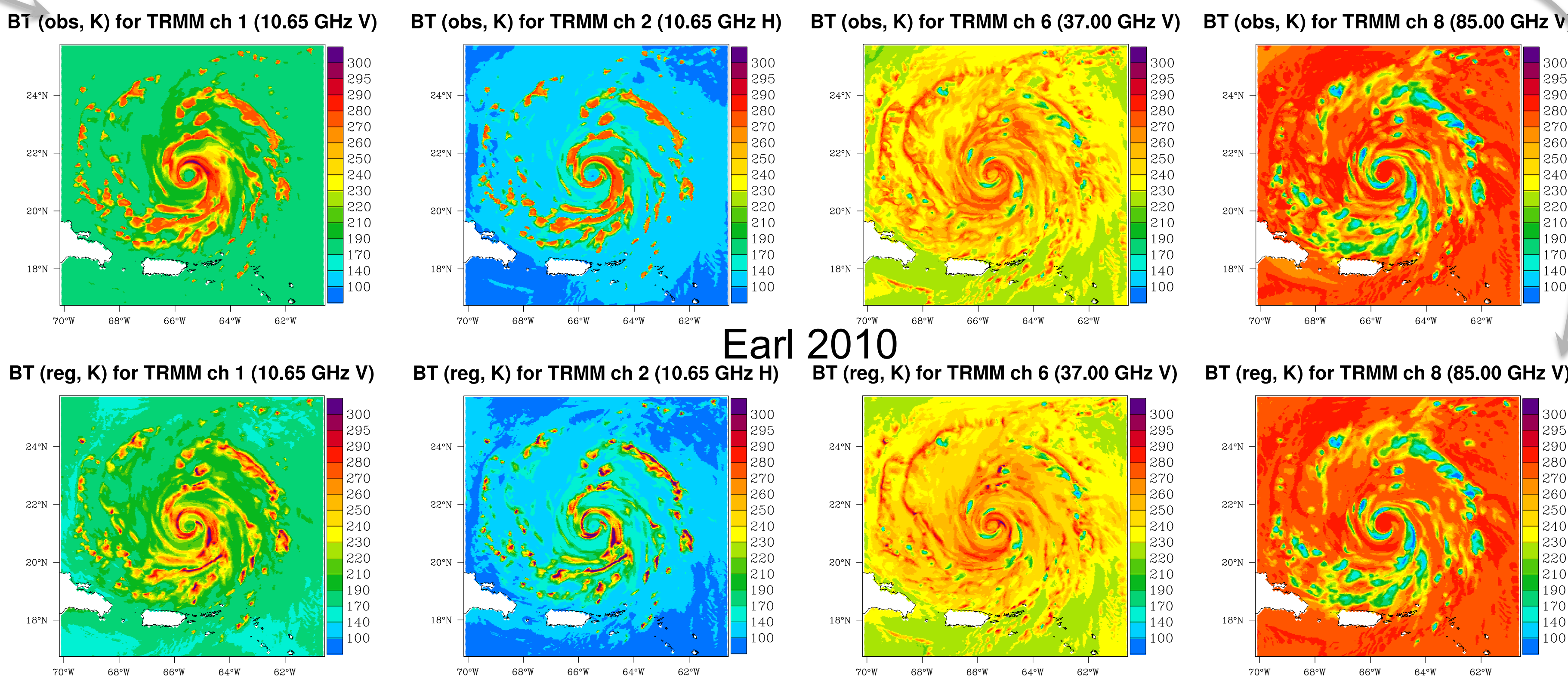
$$T_1'' = H_1(x_1'', x_2'', x_3'')$$
$$T_2'' = H_2(x_1'', x_2'', x_3'')$$
$$T_3'' = H_3(x_1'', x_2'', x_3'')$$

are not so nonlinear:

First part of step 3: T_i'' (vertical) vs x_i'' (horizontal)



First part of step 3: compare the actual T_b with approximates using x''



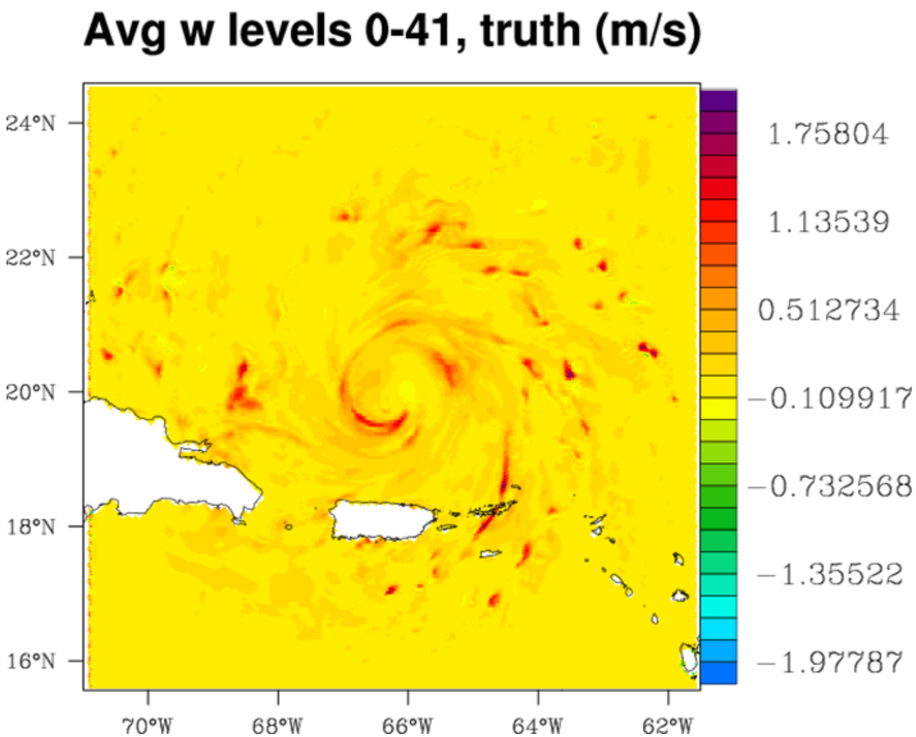
- Still not using the nonlinear representation of the observations (just the 3 combos of variables defined on slide 14 that maximize the linear correlation with corresponding combos of brightness temperatures)
- main problem: at the lowest and highest extremities of the ranges (high and low T_b)
- Let's test the performance of the non-linear differentiable expression for H

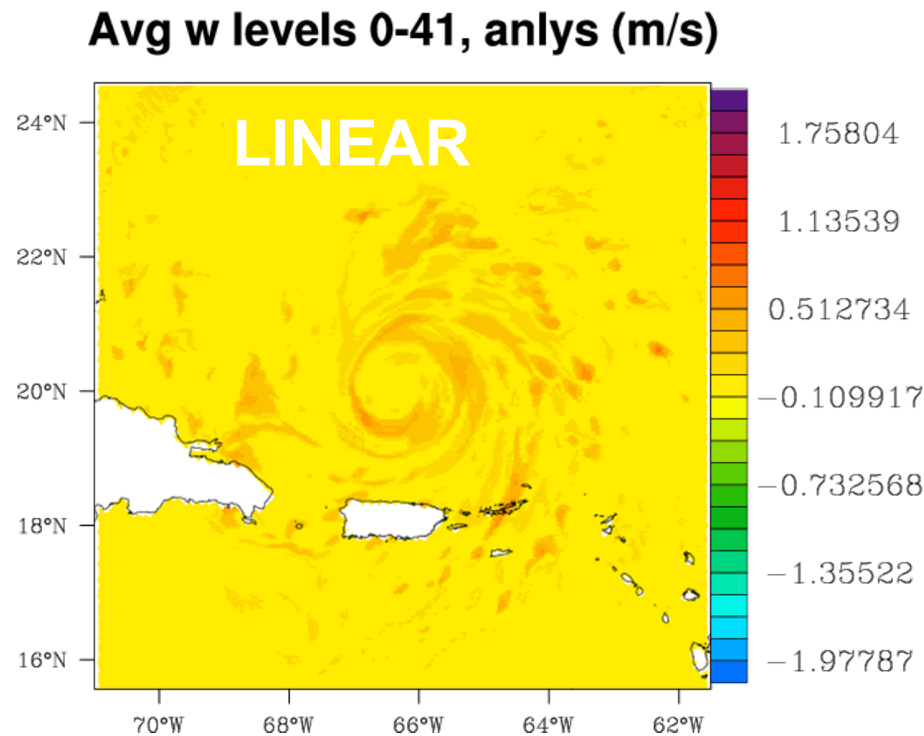
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma\ T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





vertical component of wind

Second part of step 3: use nonlinear expression

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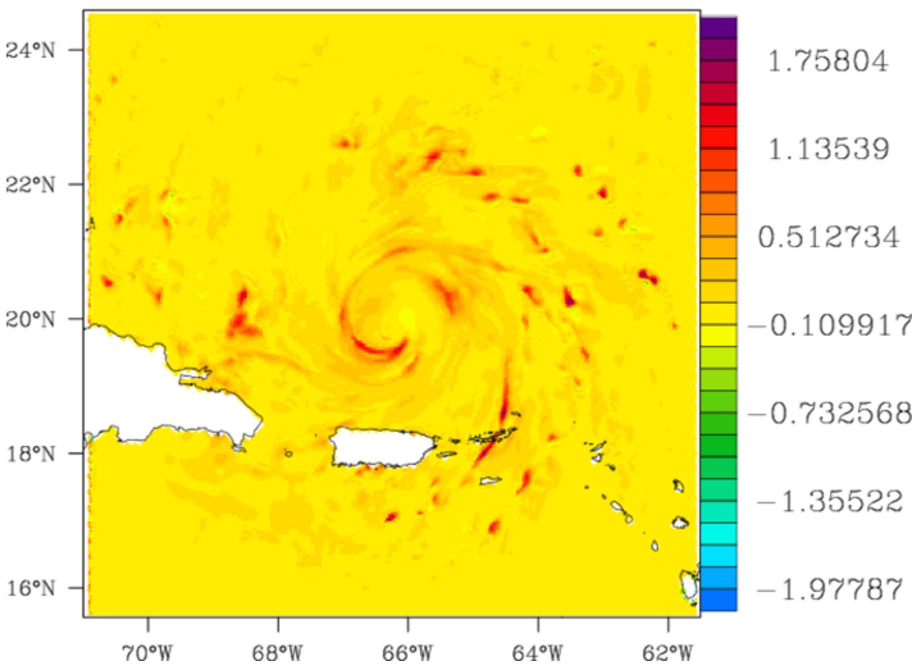
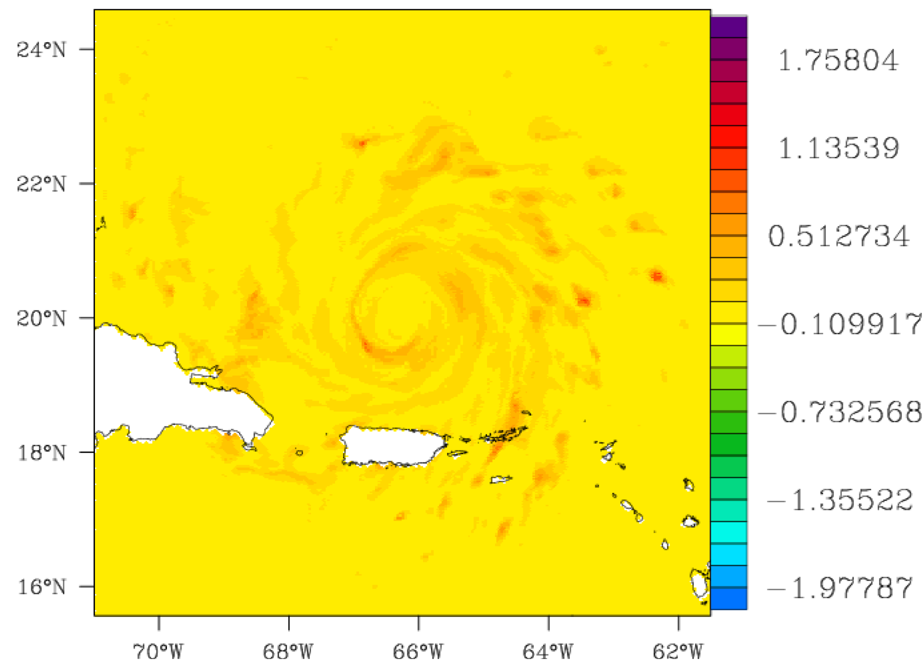
Having started with a horizontally uniform background,
each variable having the global mean value at that height level:

Avg w levels 0-41, truth (m/s)



Avg w levels 0-41, anlys (m/s)



vertical component of wind

Second part of step 3: use nonlinear expression

$T_i'' \sim x_i''$

So let's try an assimilation using this observation operator:
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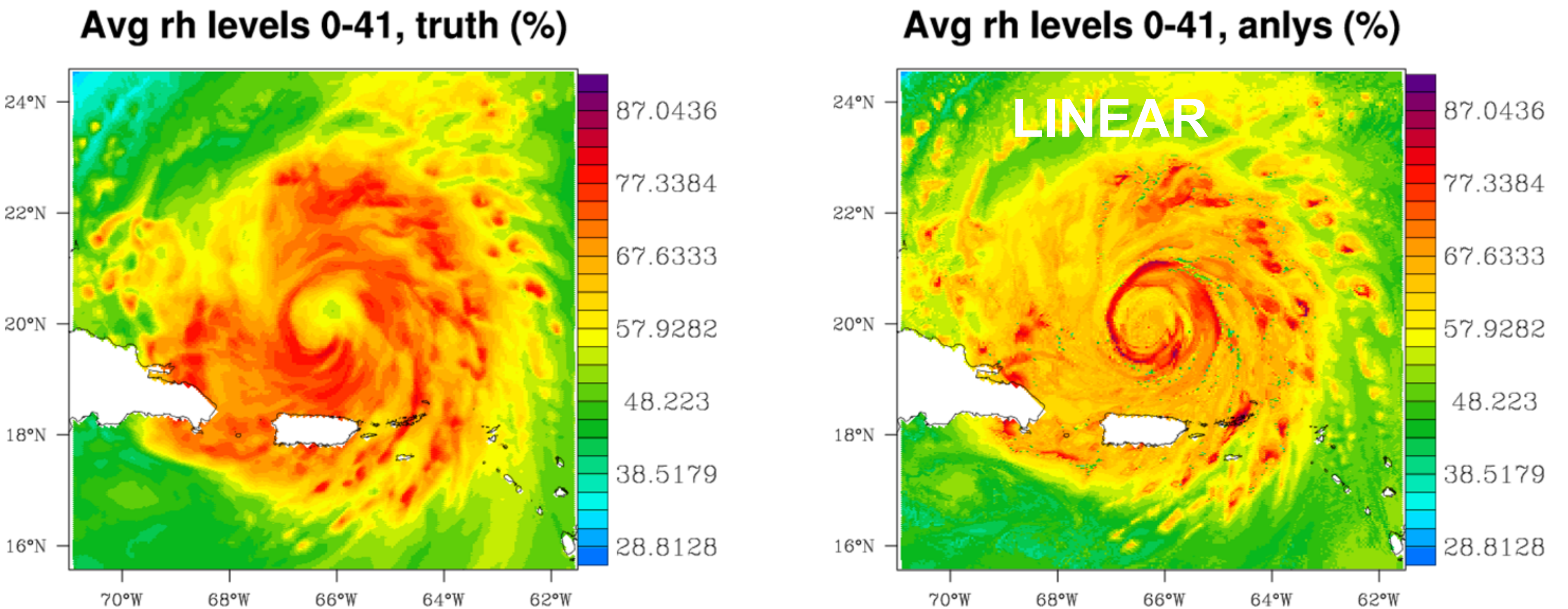
Having started with a horizontally uniform background, each variable having the global mean value at that height level:



water vapor

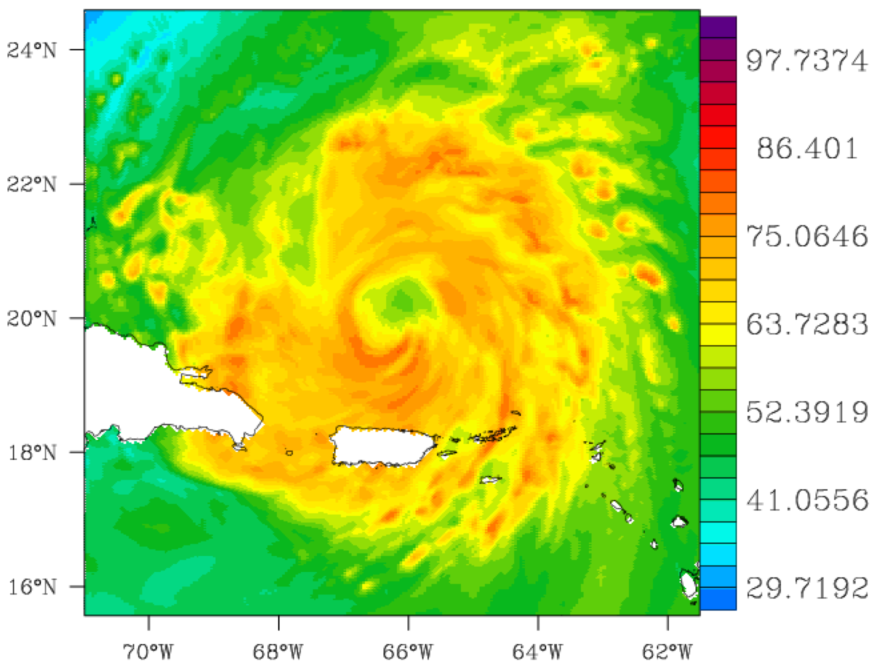
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \quad T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

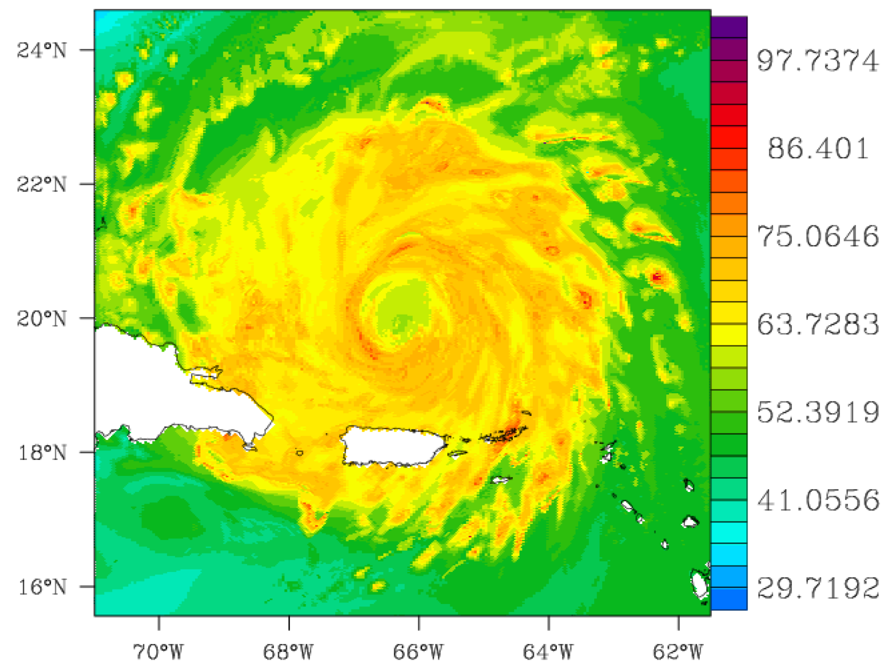
So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:

Avg rh levels 0-41, truth (%)



Avg rh levels 0-41, anlys (%)



water vapor

Second part of step 3: use nonlinear expression

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So let’s try an assimilation using this observation operator:
(start with 1D-var – not quite a “retrieval”, because of the covariances)

Having started with a horizontally uniform background,
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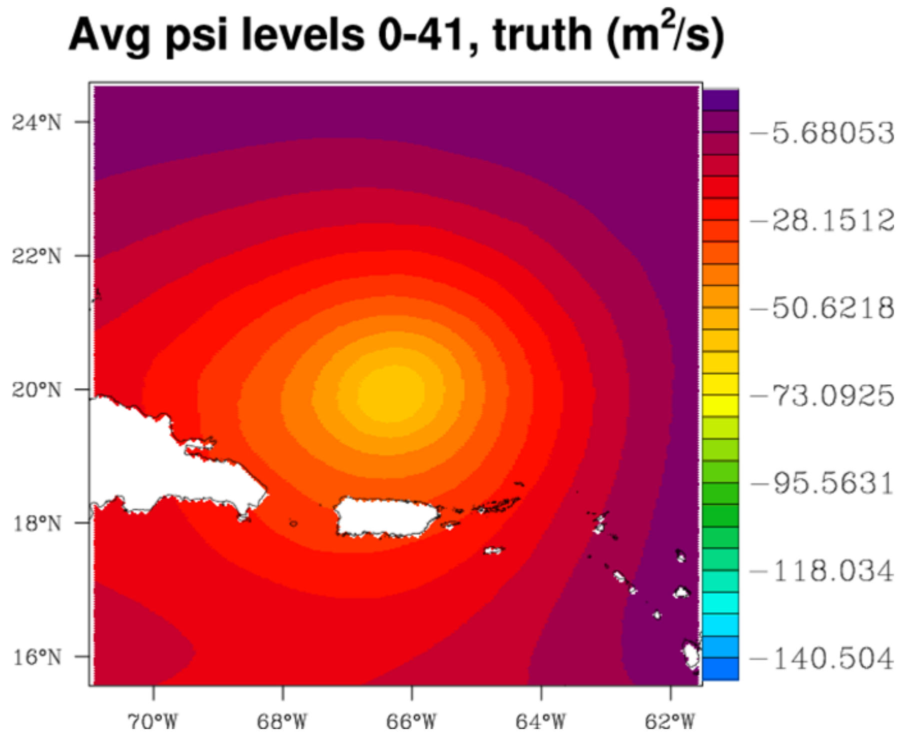
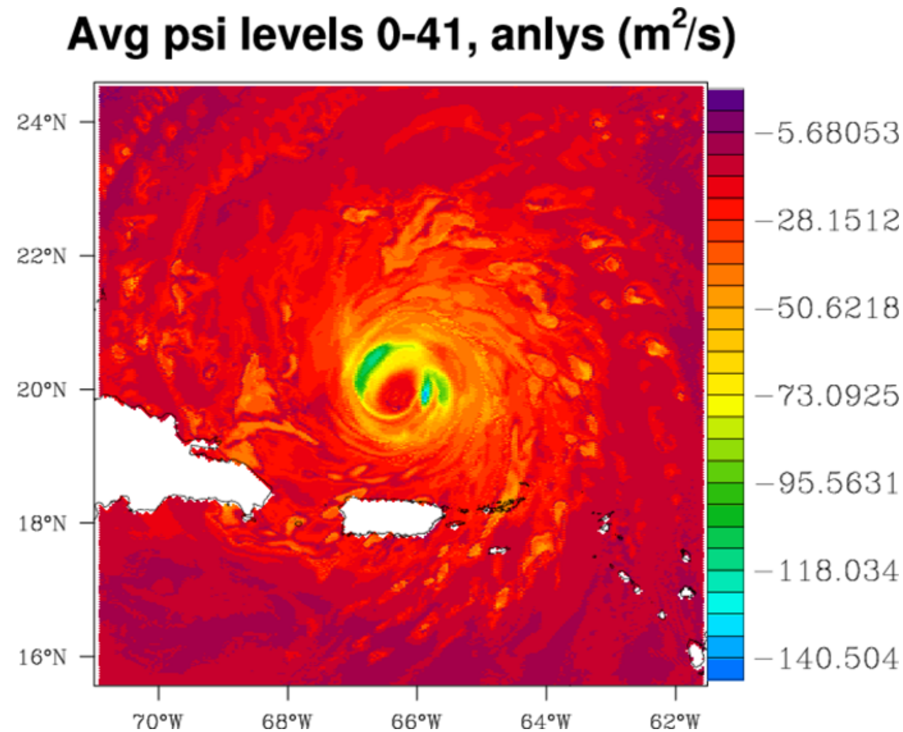




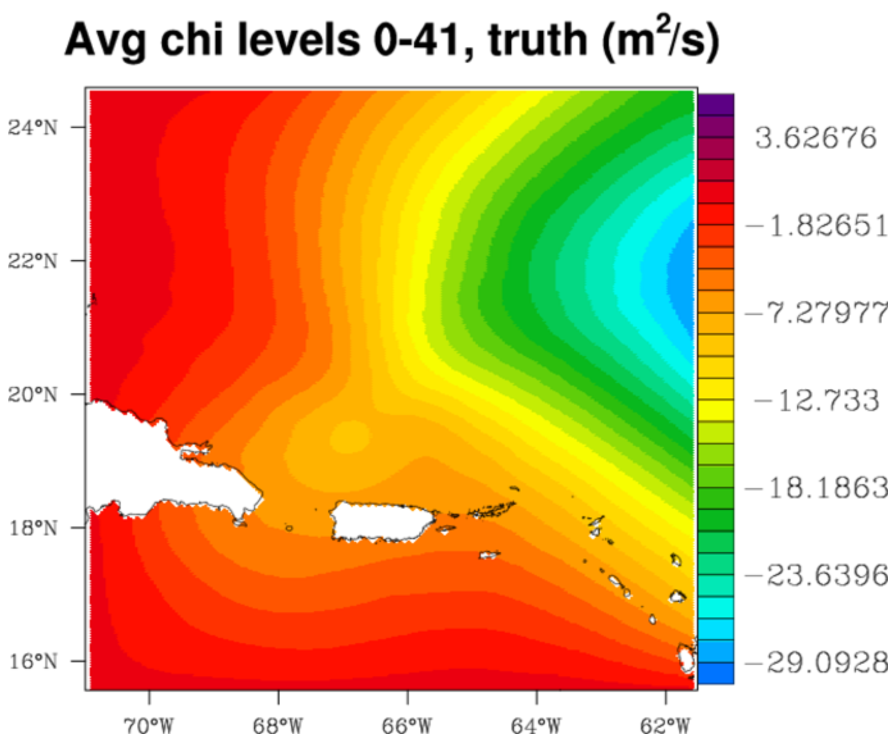
stream function

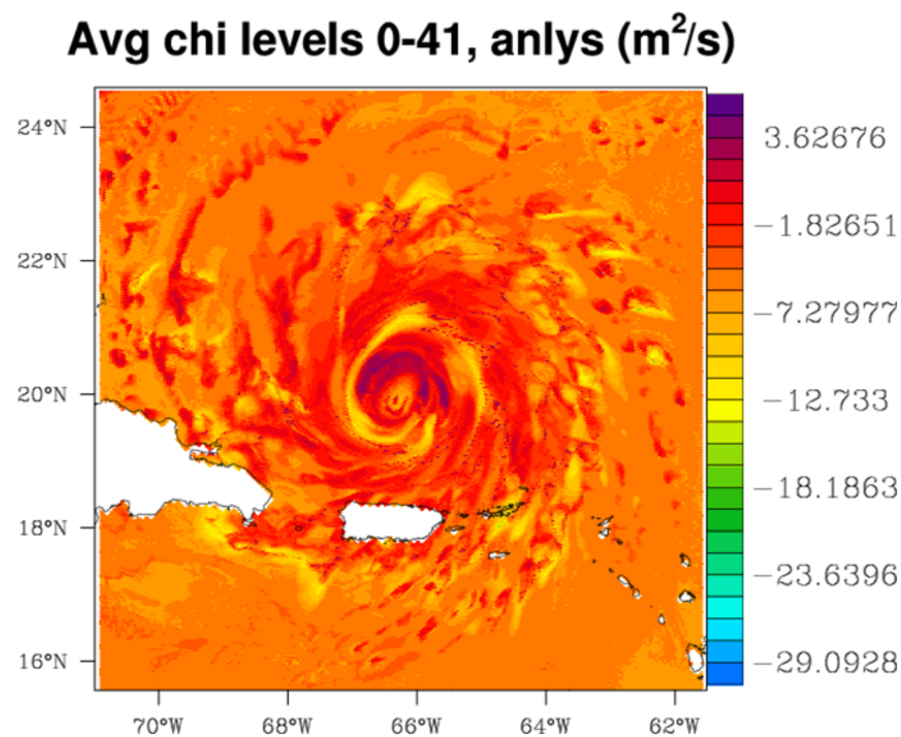
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \;\; T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





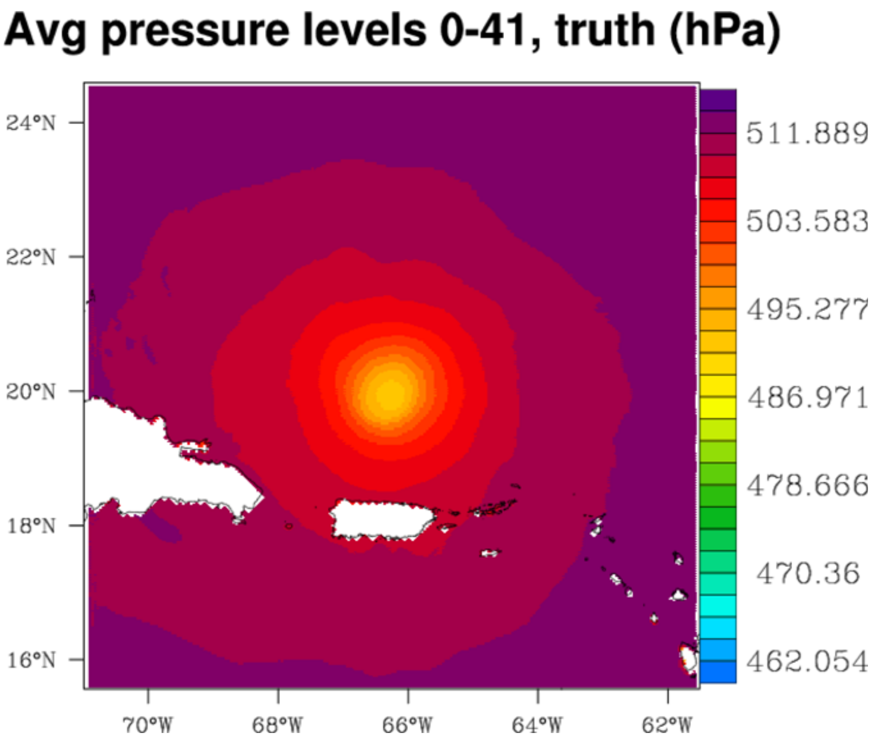
potential function

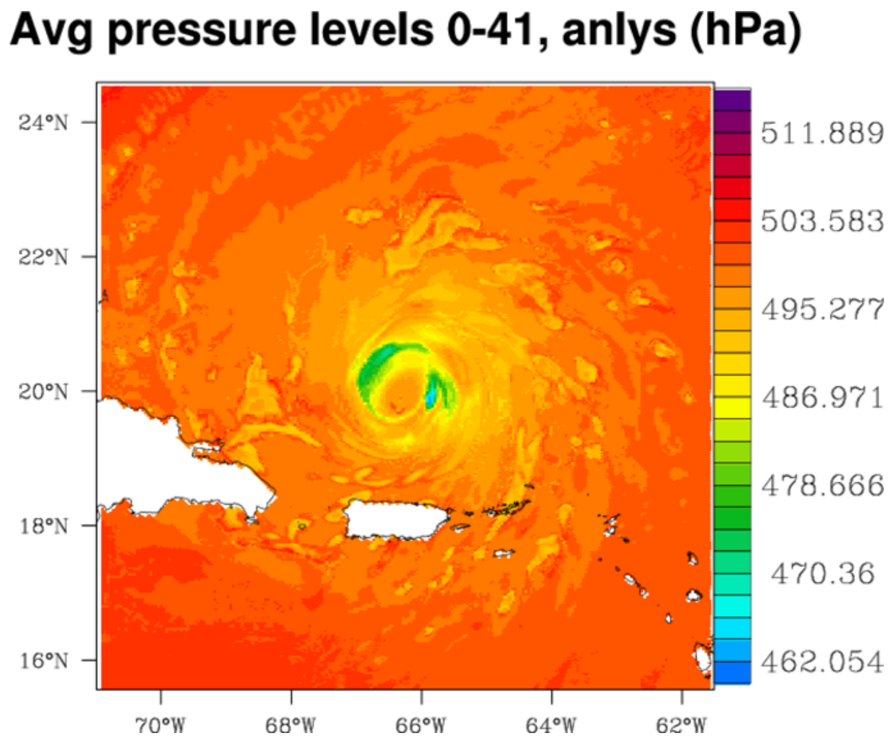
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \quad T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





pressure

Second part of step 3: use nonlinear expression

$T_i'' \sim x_i''$

So let's try an assimilation using this observation operator:
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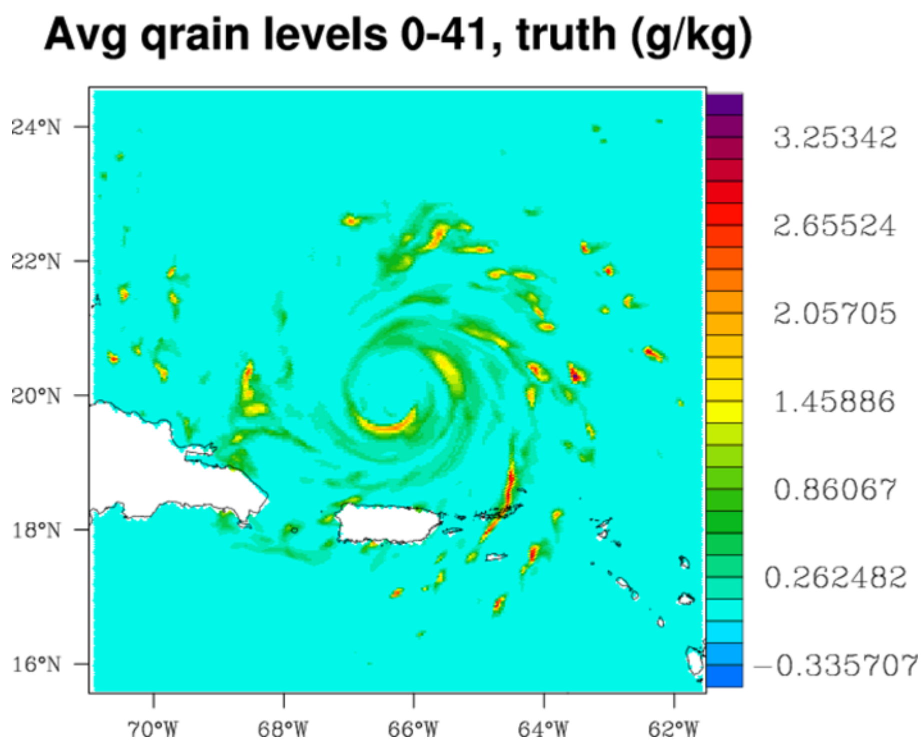
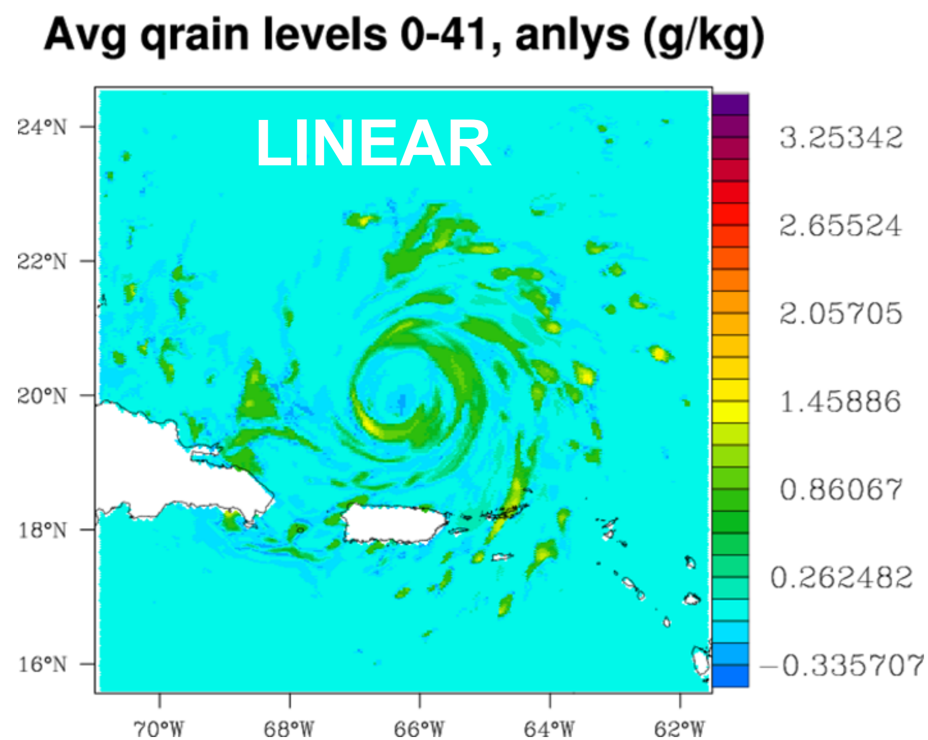
Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





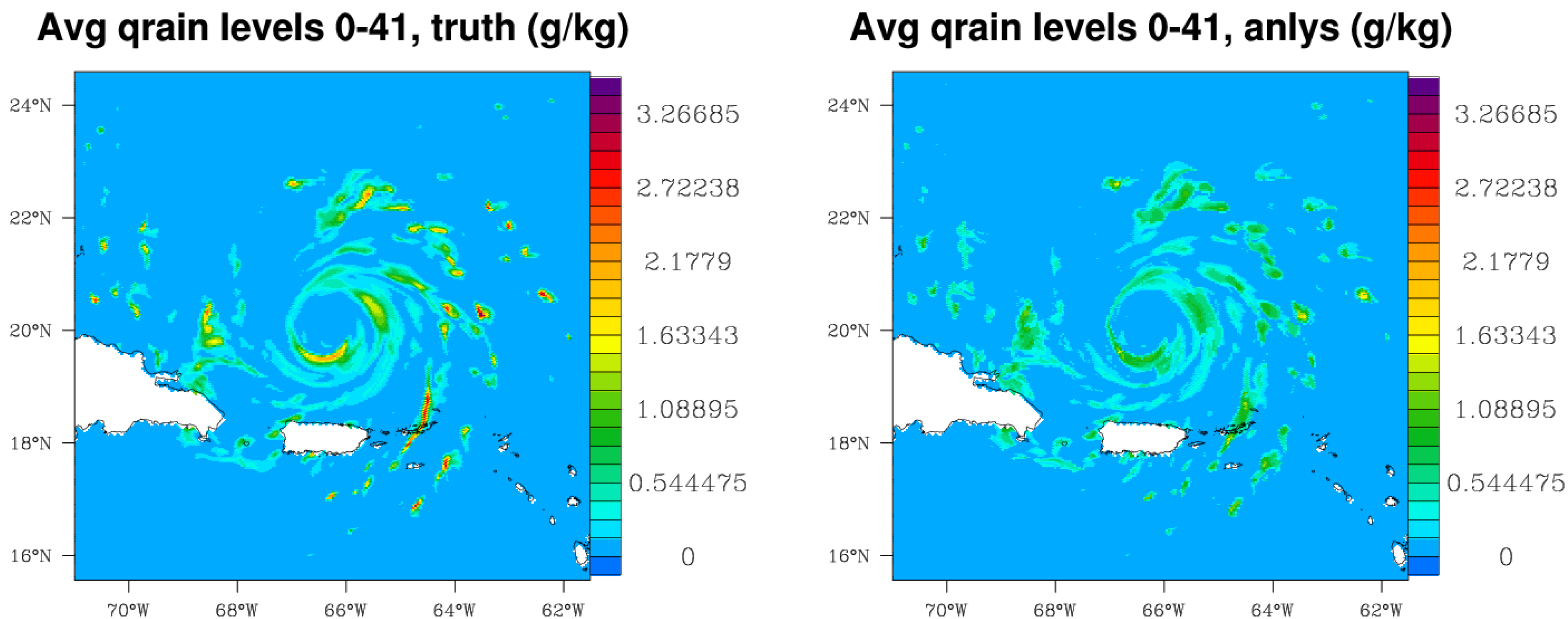
RAIN

Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \; T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:



RAIN

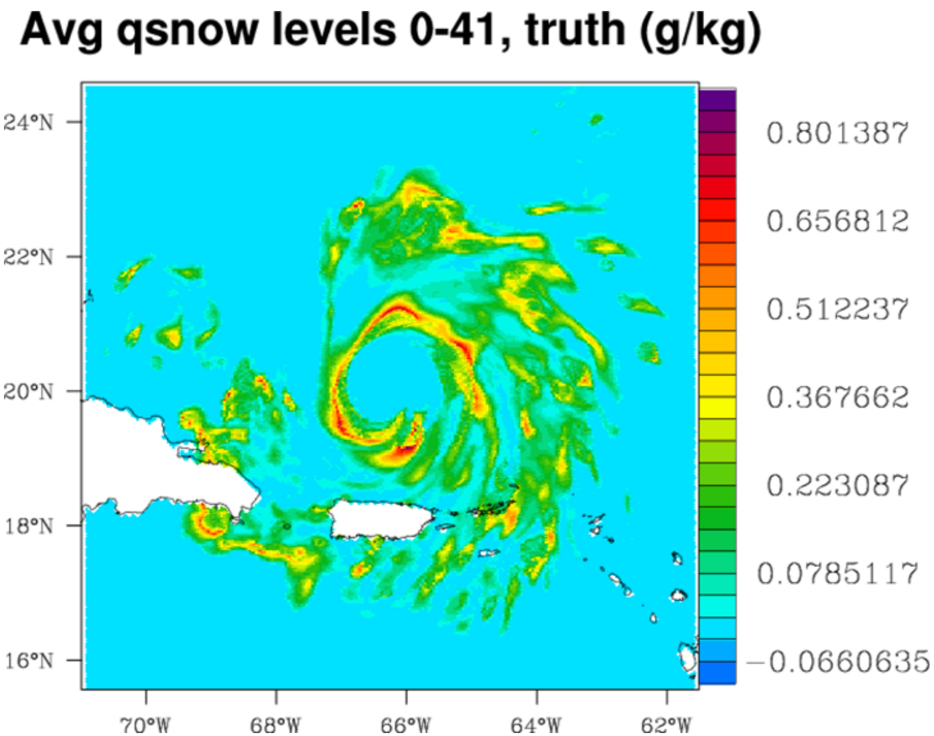
Second part of step 3: use nonlinear expression

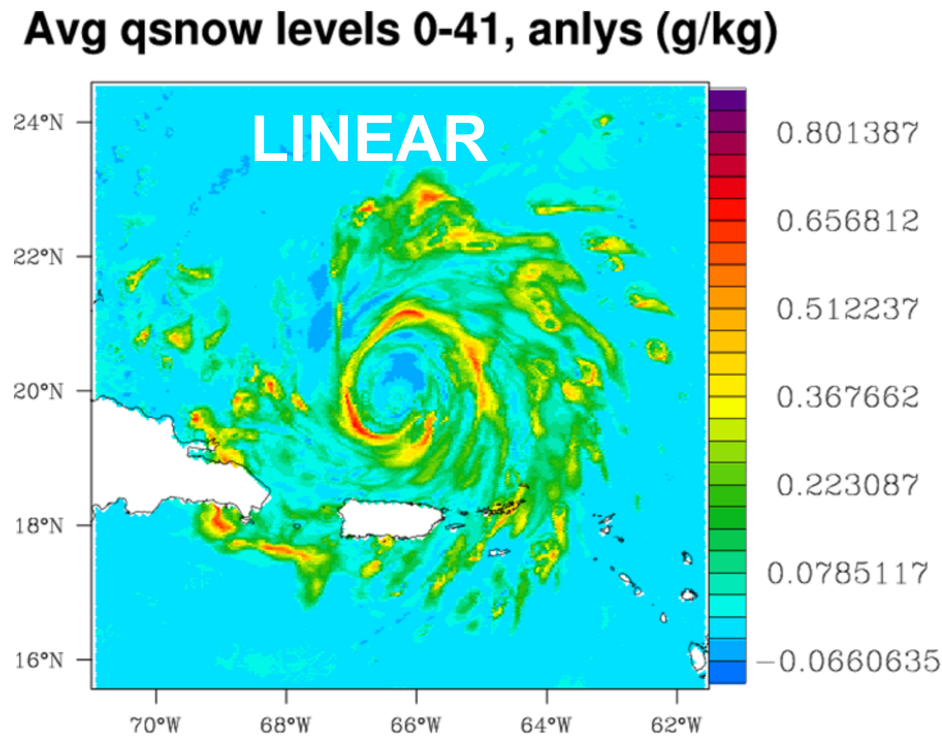
$T_i'' \sim x_i''$

So let's try an assimilation using this observation operator:

(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:





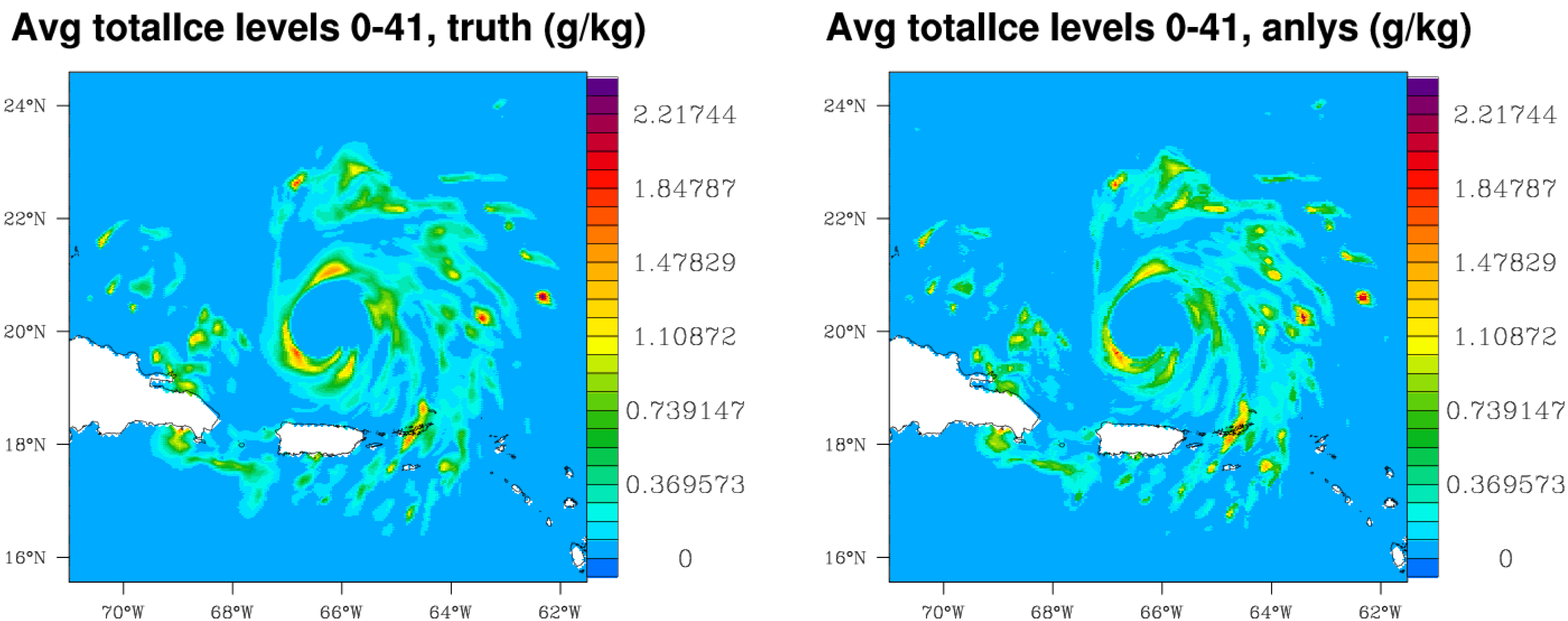
SNOW

Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \; T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:



TOTAL ICE

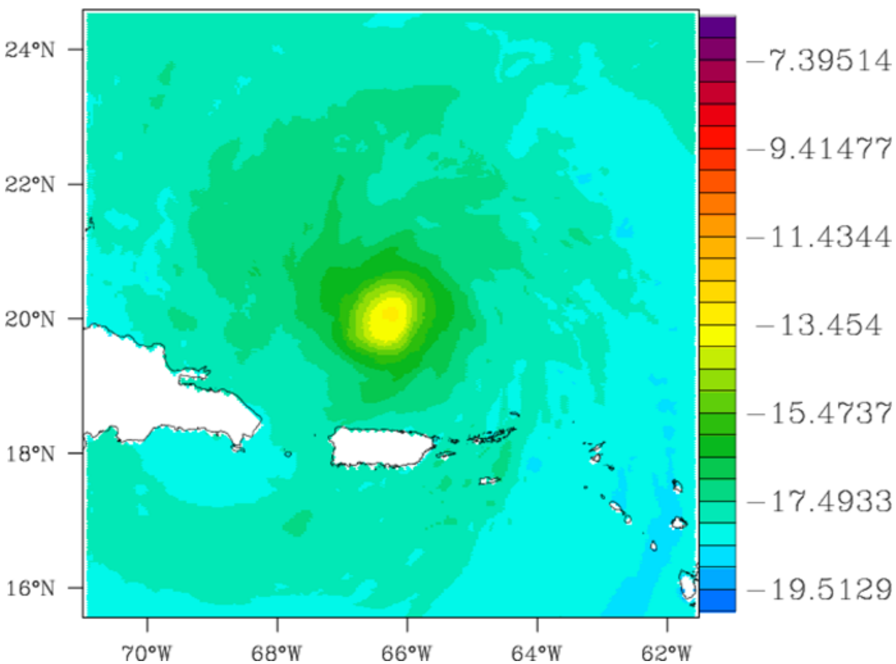
Second part of step 3: use nonlinear expression

$T_i'' \sim x_i''$

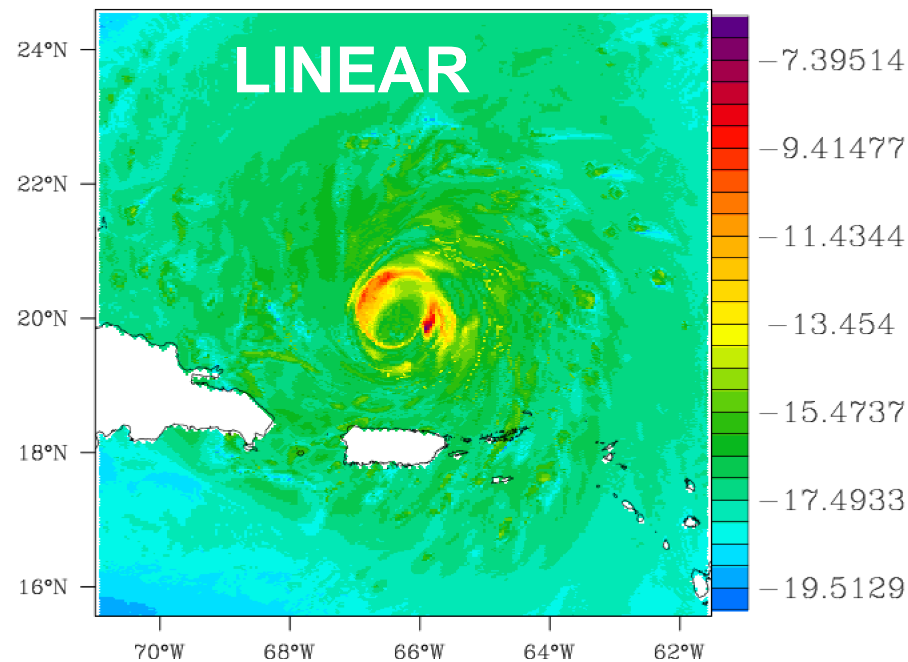
So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:

Avg temp levels 0-41, truth (C)



Avg temp levels 0-41, anlys (C)



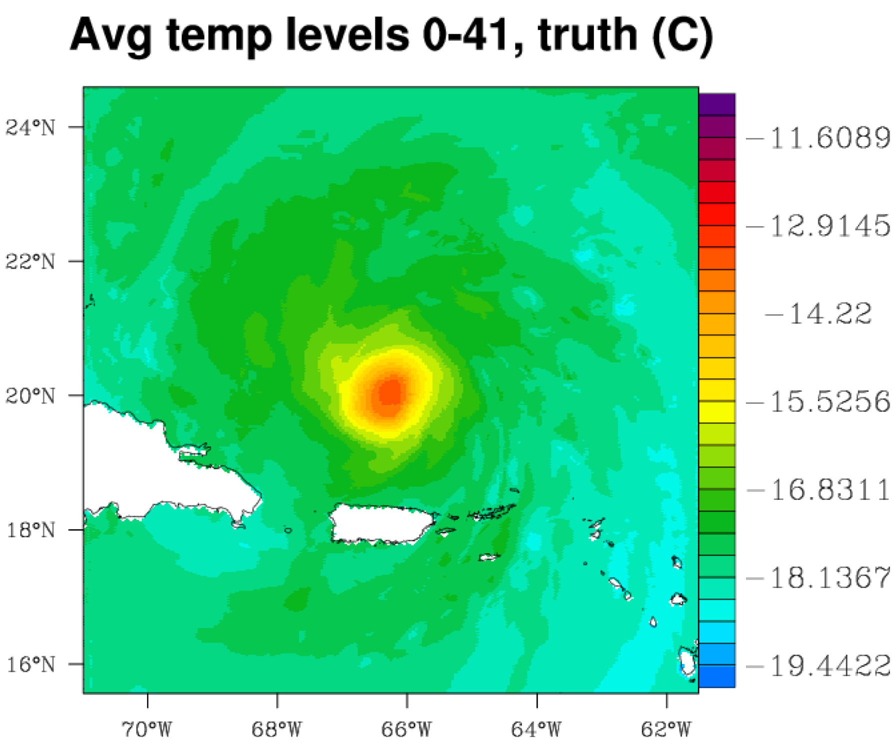
temperature

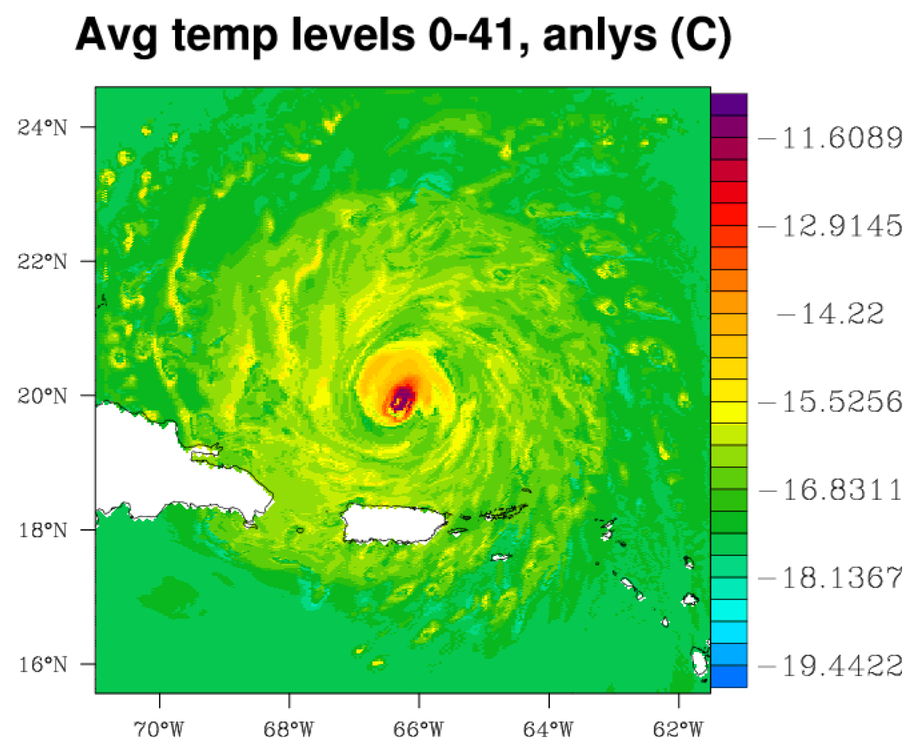
Second part of step 3: use nonlinear expression

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \quad T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

So let's try an assimilation using this observation operator:
(start with 1D-var – not quite a "retrieval", because of the covariances)

Having started with a horizontally uniform background, each variable having the global mean value at that height level:





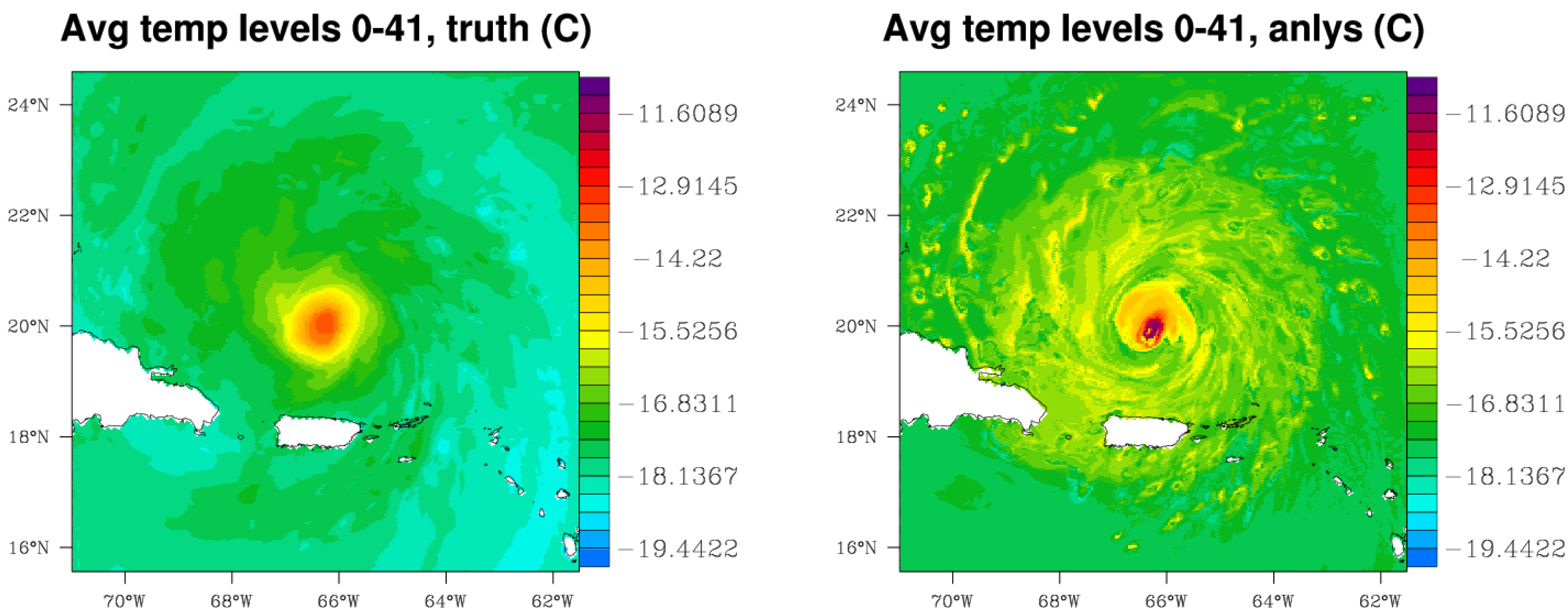
temperature

Look Ma!
a warm core!

Is this for real?

Can we really reconstruct most of the hurricane from window μwave??

Can we estimate vertical wind, and temperature anomaly, directly from the window-channel passive microwave (SSMIS, AMSR, TMI)???



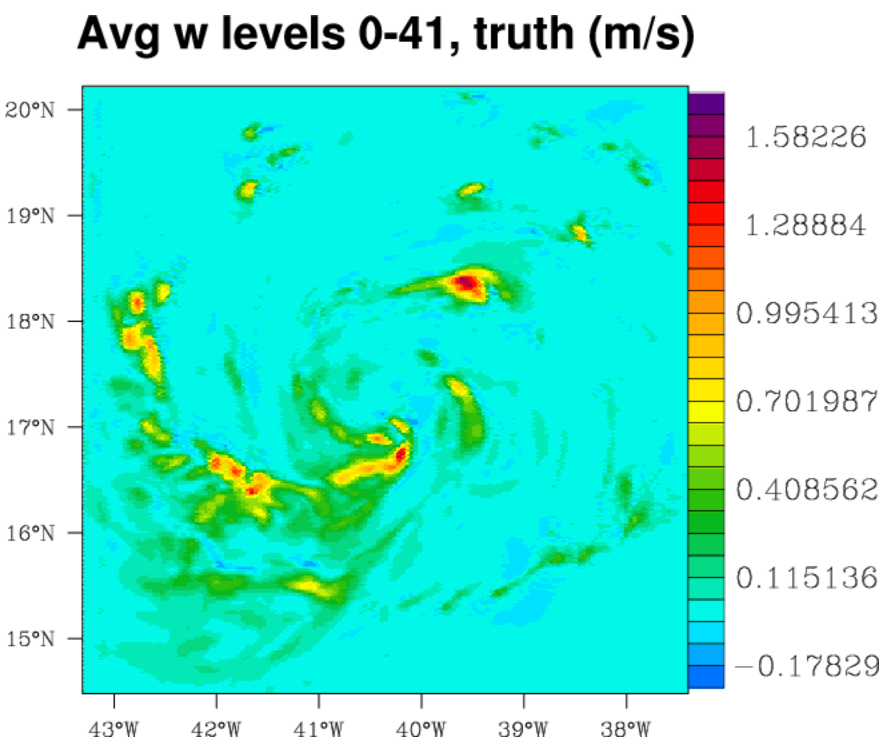
temperature

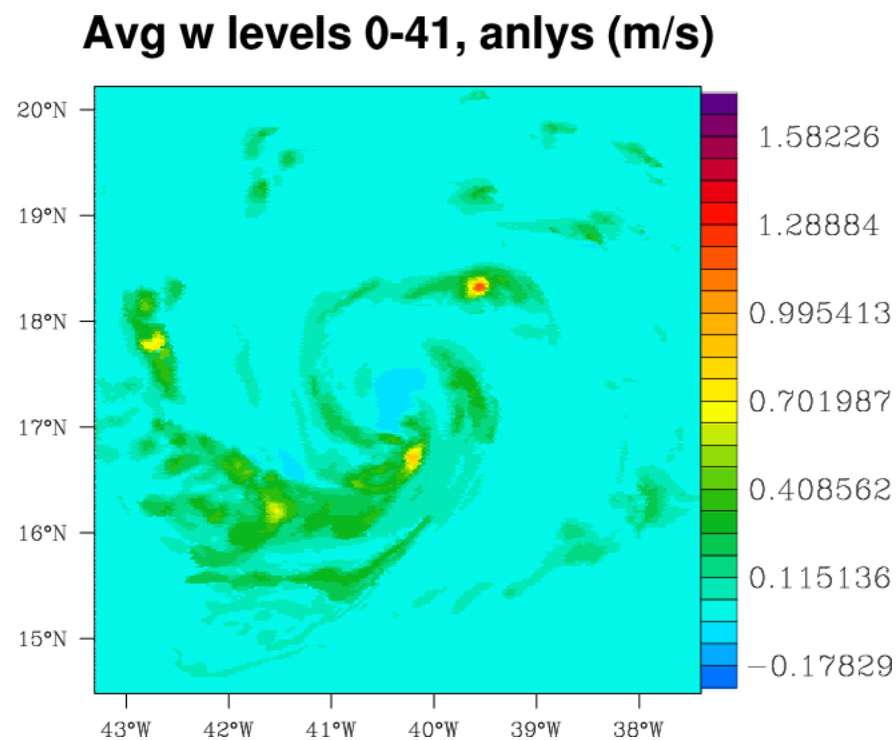
Look Ma!
a warm core!

Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





vertical component of wind

Try the exact same operator, derived from Earl, on Igor:

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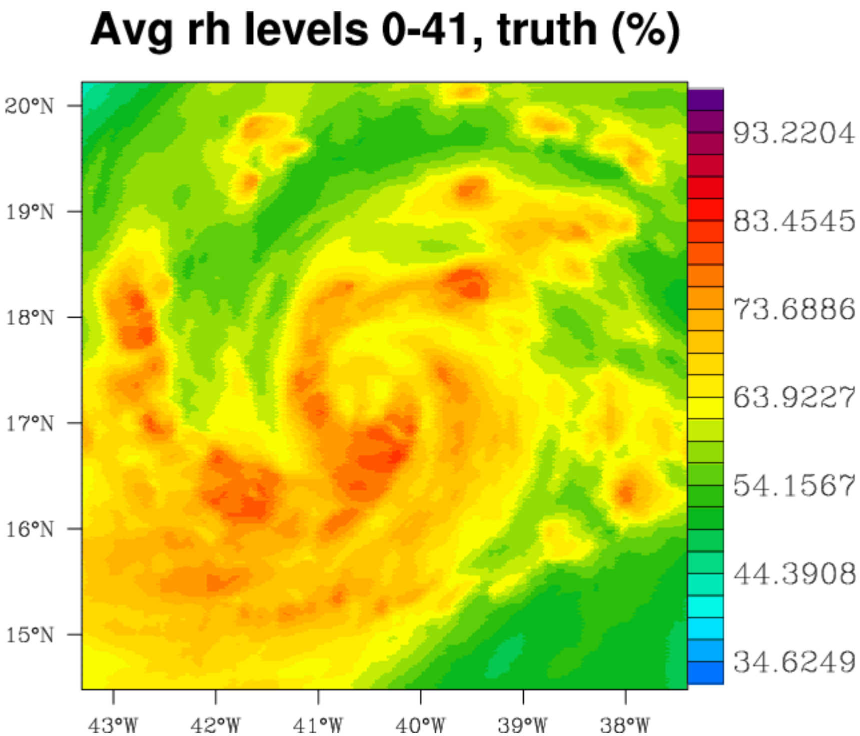

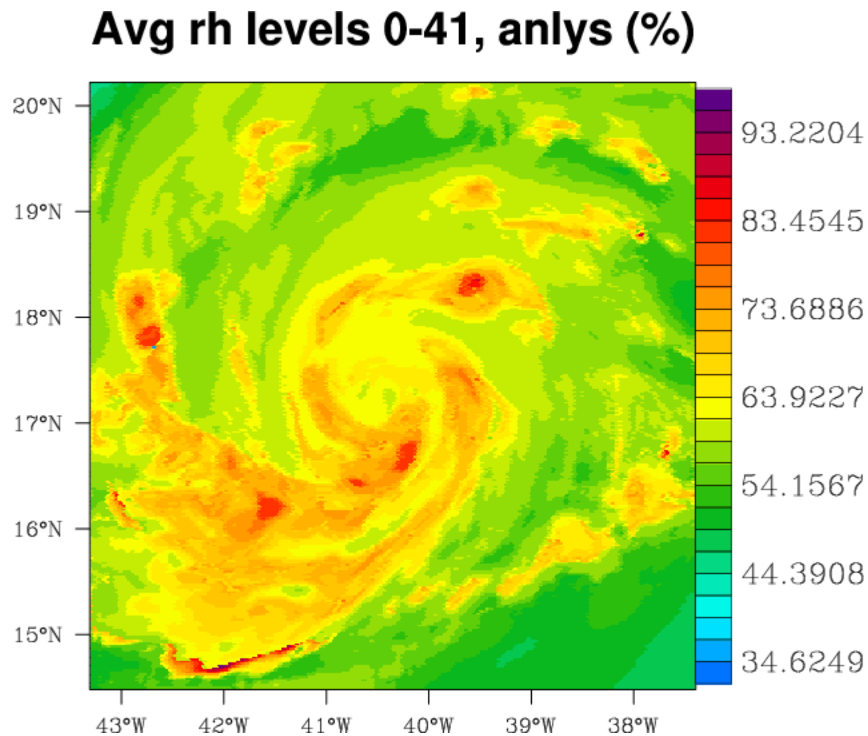

water vapor

Try the exact same operator, derived from Earl, on Igor:

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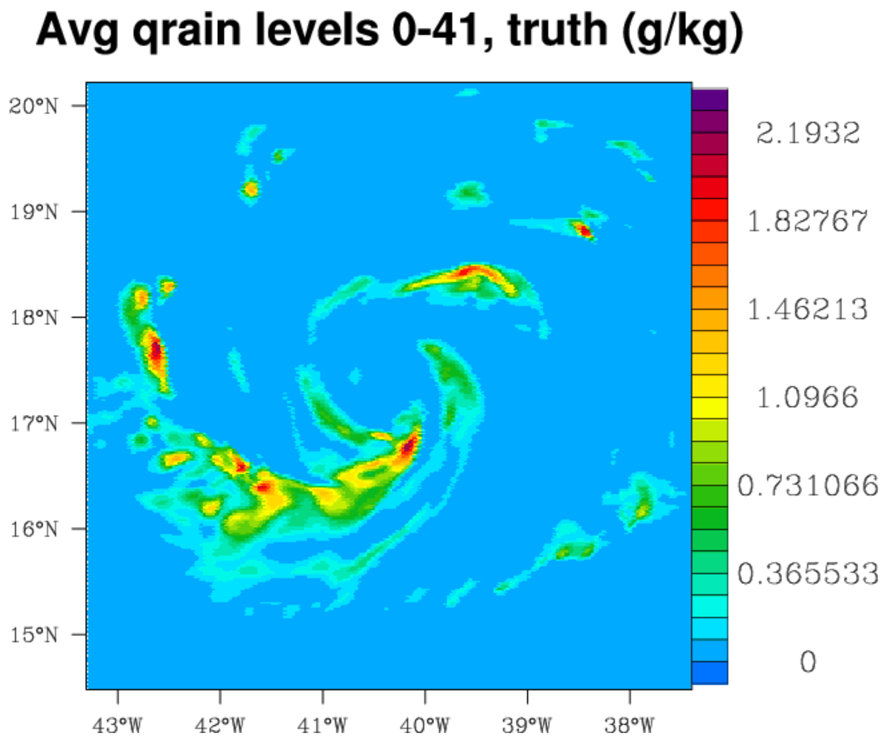
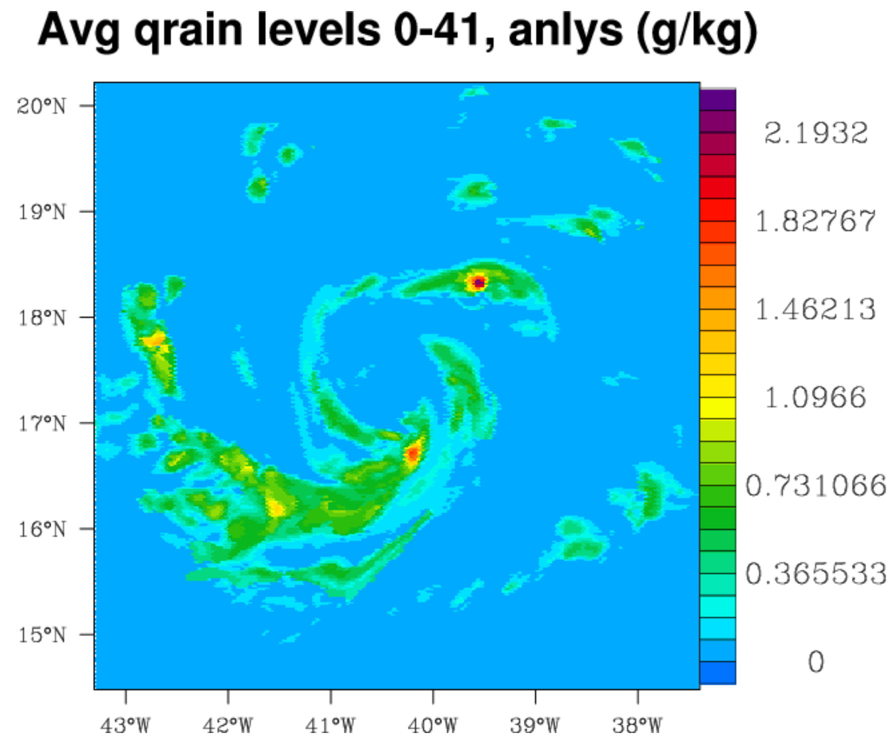


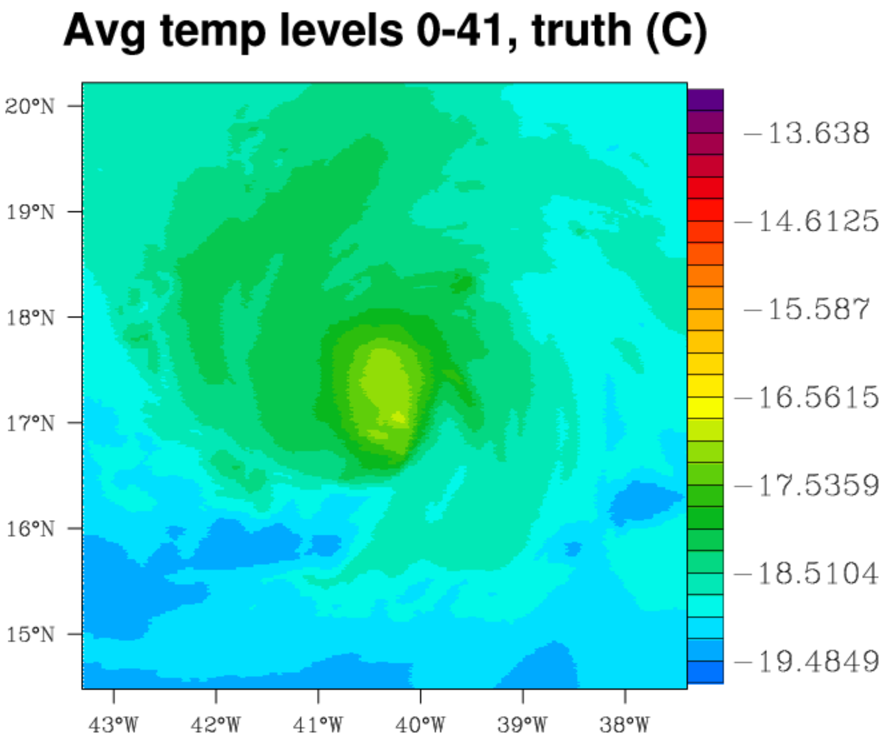


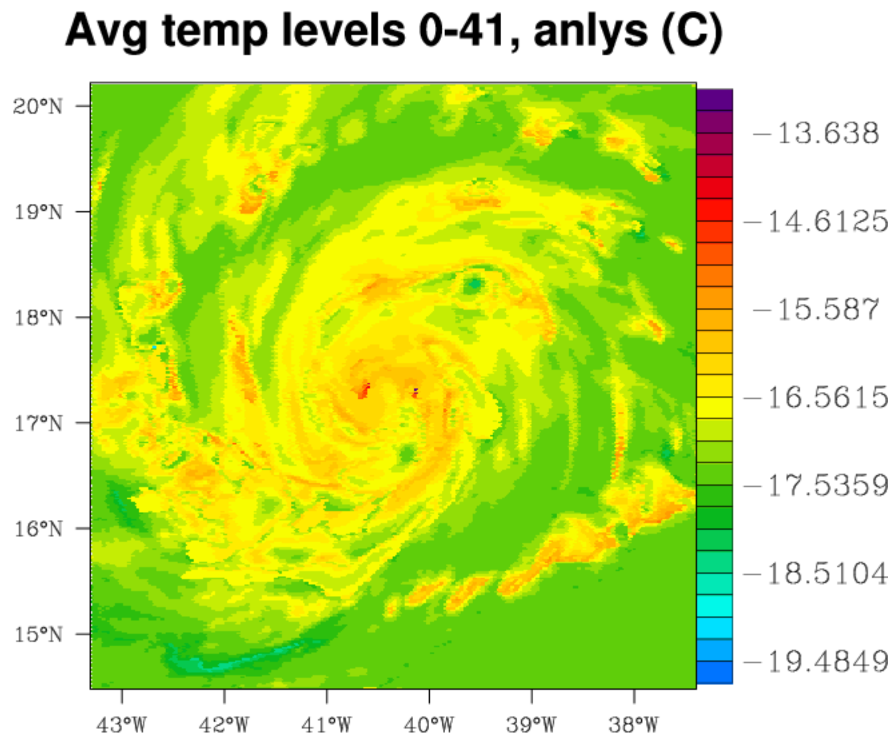
rain

Try the exact same operator, derived from Earl, on Igor:

assimilation using this observation operator:

Having started with a horizontally uniform background,
each variable having the global mean value at that height level:





temperature

So far, we have let the math define the transform variables:

$$T_1'' = H_1(x_1'', x_2'', x_3'')$$
$$T_2'' = H_2(x_1'', x_2'', x_3'')$$
$$T_3'' = H_3(x_1'', x_2'', x_3'')$$

Why not subjectively inject some physics, and impose different transform variables, dictated by expectation:

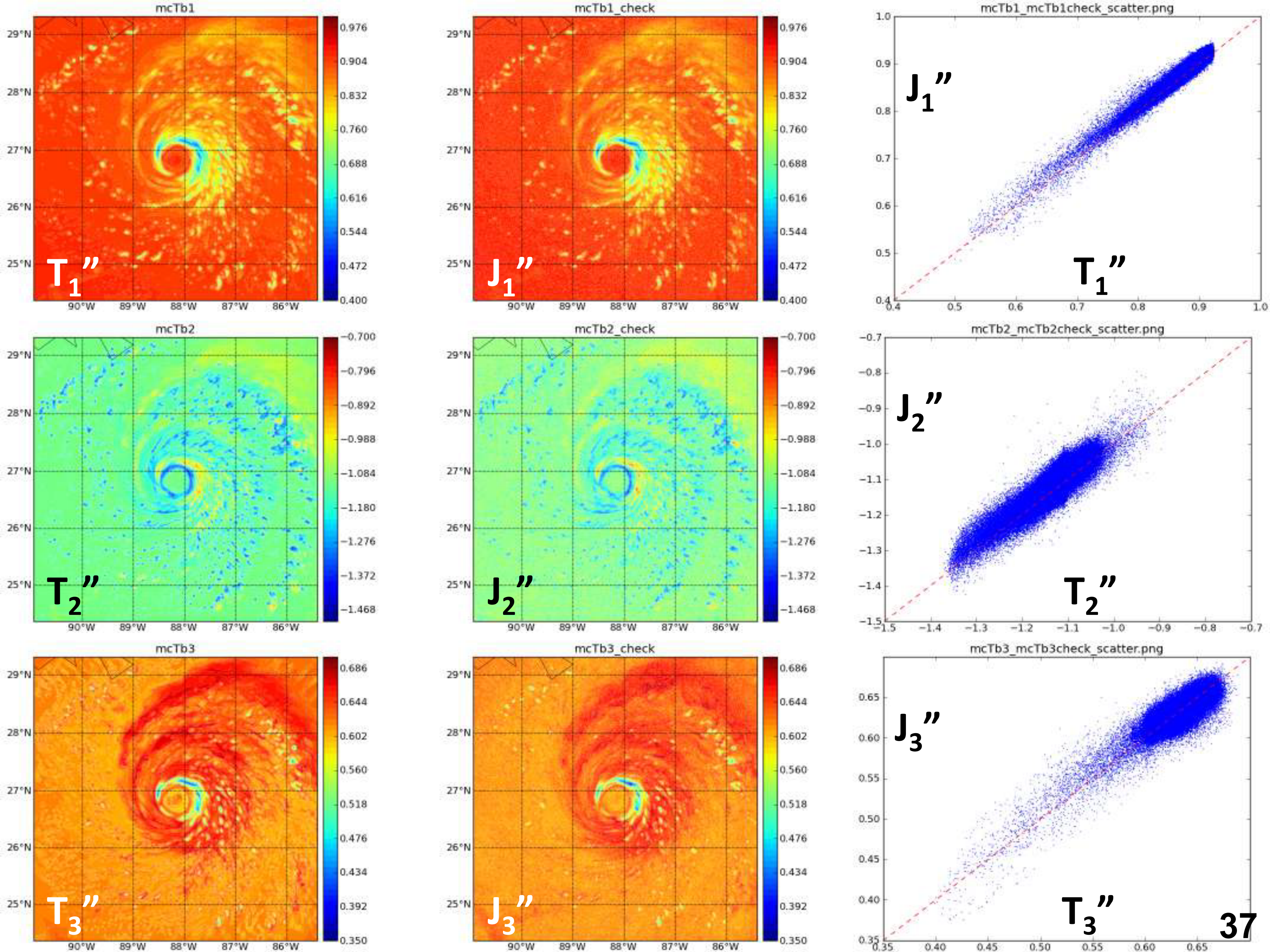
$$T_1'' = J_1(x_1'', x_{1wv}', x_{1rain}')$$
$$T_2'' = J_2(x_2'', x_{1wv}', x_{1rain}')$$
$$T_3'' = J_3(x_3'', x_{1wv}', x_{1rain}')$$

i.e. instead of

$$T_i''(x_1'', x_2'', x_3'') = \Sigma \ T_i''^{(n)} \exp(-[x_1''-x_1''^{(n)}]^2 -[x_2''-x_2''^{(n)}]^2 -[x_3''-x_3''^{(n)}]^2)$$

use

$$T_i''(x_1'', x_{1wv}', x_{1rain}'') = \Sigma \ T_i''^{(n)} \exp(-[x_i''-x_i''^{(n)}]^2 -[x_{1wv}'-x_{1wv}'^{(n)}]^2 -[x_{1rain}'-x_{1rain}'^{(n)}]^2)$$


mcTb1
mcTb1_check
mcTb1_mcTb1check_scatter.png
T1”
J1”
0.976
0.904
0.832
0.760
0.688
0.616
0.544
0.472
0.400
mcTb2
mcTb2_check
mcTb2_mcTb2check_scatter.png
T2”
J2”
−0.700
−0.796
−0.892
−0.988
−1.084
−1.180
−1.276
−1.372
−1.468
mcTb3
mcTb3_check
mcTb3_mcTb3check_scatter.png
T3”
J3”
0.686
0.644
0.602
0.560
0.518
0.476
0.434
0.392
0.350
29°N
28°N
27°N
26°N
25°N
90°W
89°W
88°W
87°W
86°W

Jeff Steward is incorporating operator into HWRF EnKF DAS

Additional slides on hidden problems in microphysical representations

Implicit unrealistic correlations in hydrometeor size distributions:

$\Rightarrow$ Assume closed-form diameter distributions (e.g. exponential or Γ):

$$N(D) = N_0 \, D^{\mu} \, e^{-\Lambda D}$$

Let's try to interpret the parameters in terms of physically meaningful quantities:

$$D_m = \frac{\int D \, D^3 N(D) \, dD}{\int D^3 N(D) \, dD} = \frac{\mu + 4}{\Lambda}$$

$$q = \int \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 \rho \, N(D) \, dD = \frac{\pi}{6} \frac{\rho \, \Gamma(\mu+1) \, D_m^{\mu+1}}{(\mu+4)^{\mu+1}} \, N_0$$

$$\sigma_m = \left(\frac{\int (D - D_m)^2 \, D^3 N(D) \, dD}{\int D^3 N(D) \, dD} \right)^{1/2} = \frac{D_m}{\sqrt{\mu + 4}}$$

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In the models, typically assume N_0 constant and $\mu = 0$.

What that implies is:

$$q = \int \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \rho \; N(D) \; dD = \frac{\pi}{6}\frac{\rho \;\Gamma(\mu+1) \; D_m^{\mu+1}}{(\mu+4)^{\mu+1}} N_0$$

$$= \frac{\pi\rho}{24} N_0 \; D_m$$

Implicit unrealistic correlations in hydrometeor size distributions:

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In the models, typically assume N_0 constant and $\mu = 0$.

What that implies is:

$$D_m = \frac{24}{\pi \rho N_0} q$$

In particular,

- D_m/q = constant, and
- max(D_m)/min(D_m) = max(q)/min(q)

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What that implies is:

$$D_m = \frac{24}{\pi\rho N_0}\, q$$

In particular,

- D_m/q = constant, and
- $\max(D_m)/\min(D_m) = \max(q)/\min(q)$

But 3.5 mm / 0.5 mm $\neq$ $\max(R^{0.9})/\min(R^{0.9}) \approx$ 100 mm/hr / 0.1 mm/hr

Implicit unrealistic correlations in hydrometeor size distributions:

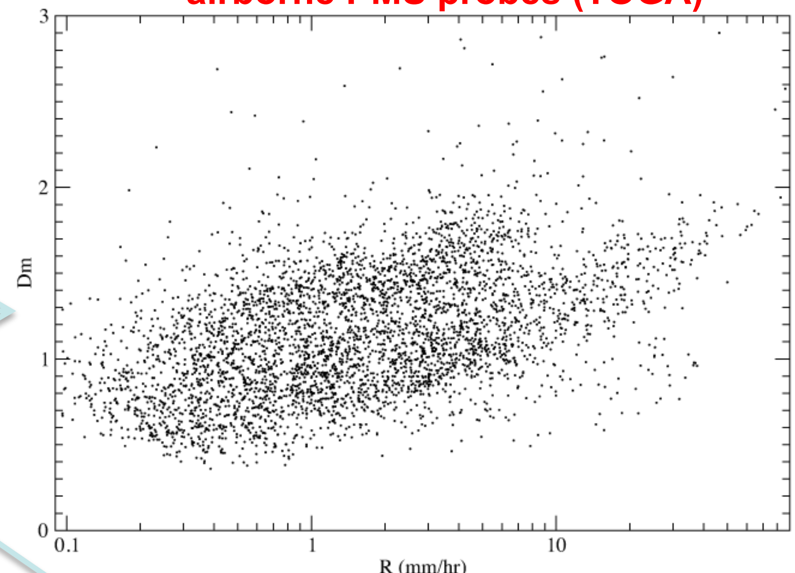
⇒ In fact, hydrometeor data suggests $D_m \sim q^{0.2}$ ± white noise

$D_m \sim q^{0.2}$ behavior in profiler data

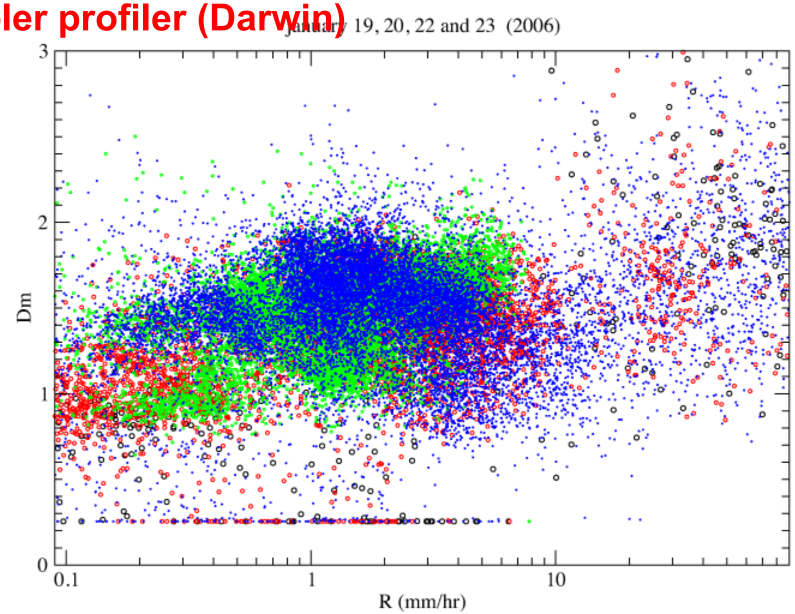
is consistent with TOGA-COARE

and Kwajex data …

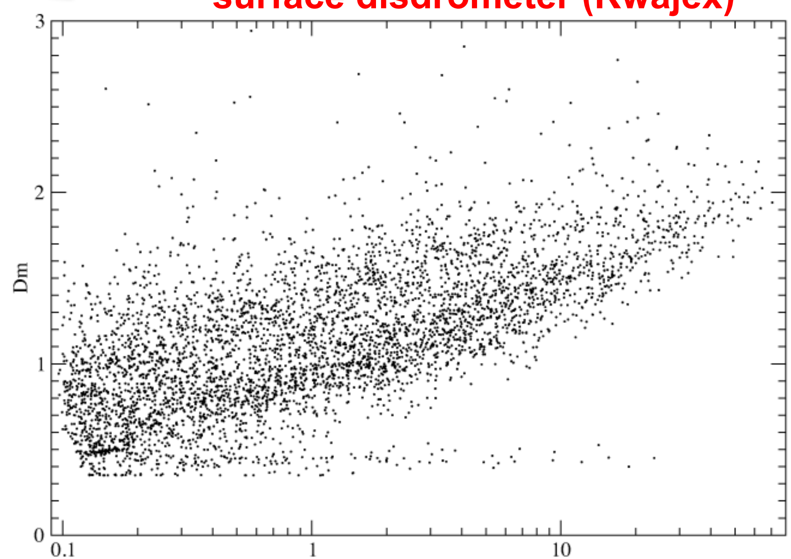
airborne PMS probes (TOGA)



Doppler profiler (Darwin)



surface disdrometer (Kwajex)



Implicit unrealistic correlations in hydrometeor size distributions:

$\Rightarrow$ Assume closed-form diameter distributions (e.g. exponential or Γ):

$$N(D) = N_0 \; D^{\mu} \; e^{-\Lambda D}$$

Fixing Λ is at least as problematic:

$$D_m = \frac{\int D \; D^3 N(D) \; dD}{\int D^3 N(D) \; dD} = \frac{\mu + 4}{\Lambda}$$

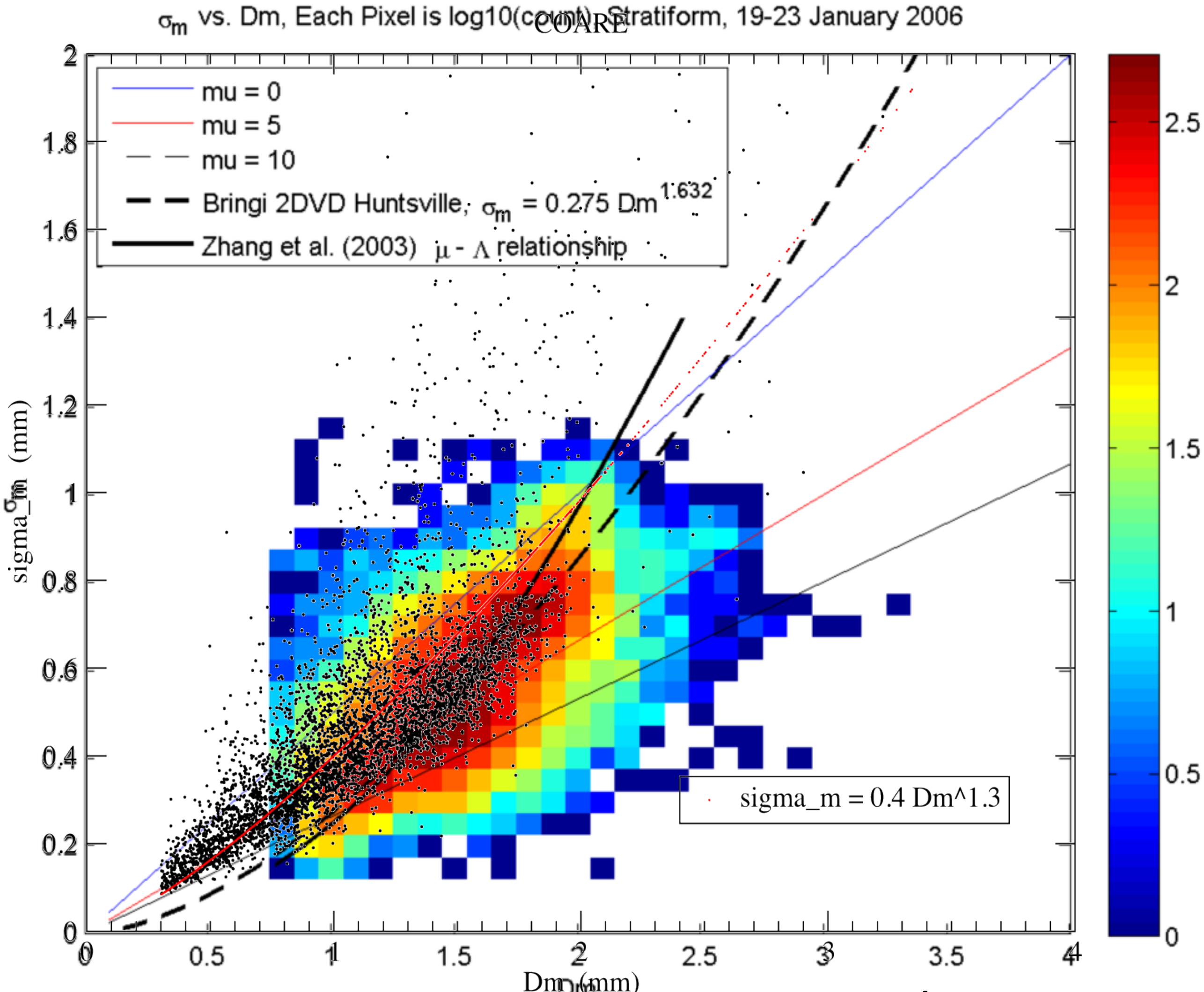
$$\sigma_m = \left(\frac{\int (D - D_m)^2 \; D^3 N(D) \; dD}{\int D^3 N(D) \; dD} \right)^{1/2} = \frac{D_m}{\sqrt{\mu + 4}}$$

The above imply:

$$D_m = \frac{D_m^2 / \sigma_m^2}{\Lambda} \;\Rightarrow\; \frac{D_m}{\sigma_m^2} = const \quad \text{, i.e.} \quad \sigma_m = const \; D_m^{0.5}$$

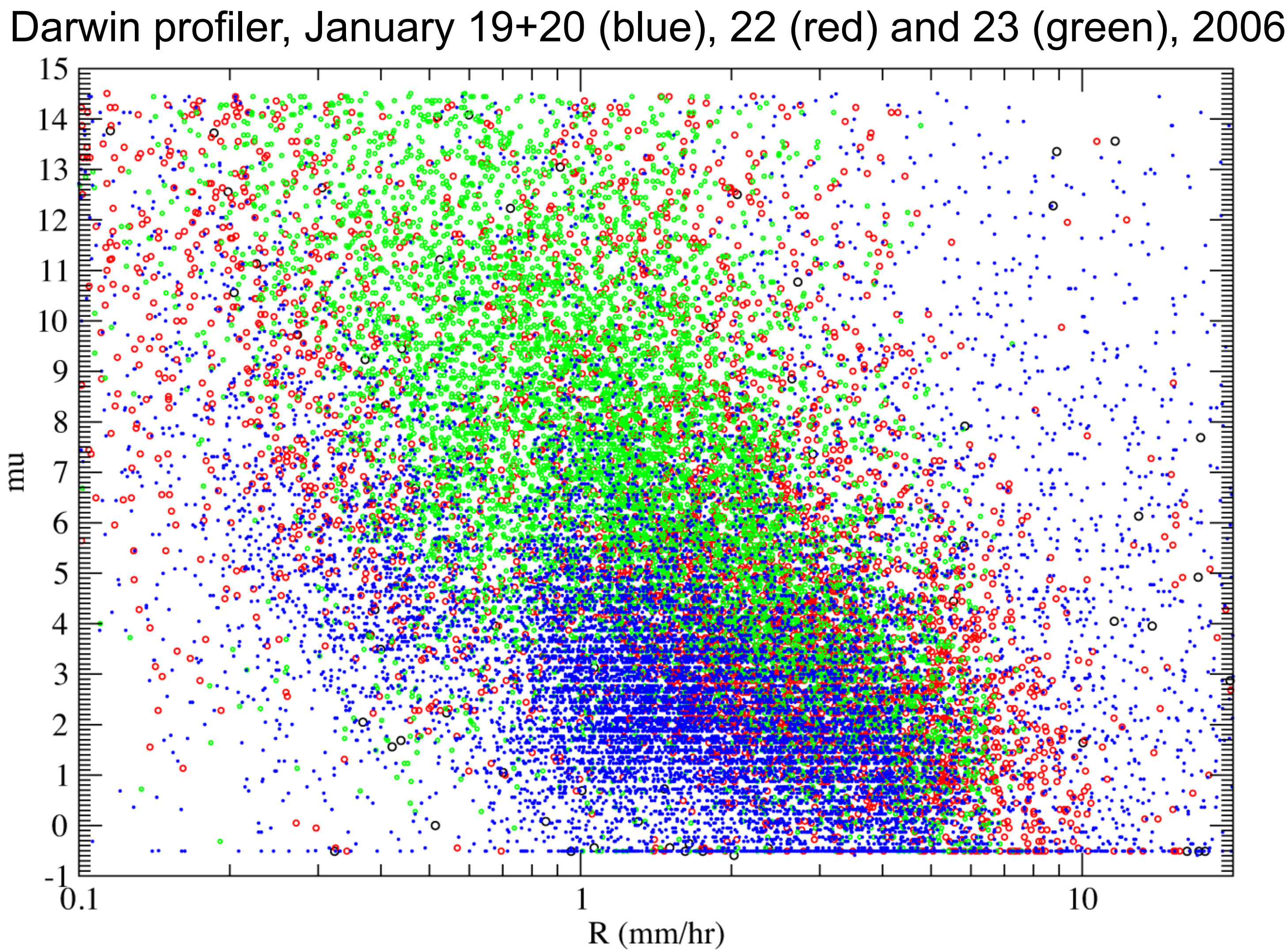
Implicit unrealistic correlations in hydrometeor size distributions:

⇒ Observations indicate: $\sigma_m = const \cdot D_m^{1.5} \pm noise$



$(\sigma_m \neq \ const \ D_m^{0.5})$

Furthermore, μ is neither 0 nor constant (& neither are N_0, Λ):



Coup de grâce: $$\frac{\partial \mu}{\partial t} + V \cdot \nabla \mu = ? \qquad \frac{\partial \sigma_m}{\partial t} + V \cdot \nabla \sigma_m = ?$$